Preface

This book has been created for the basic math student who needs a thorough review of arithmetic and pre-algebra principles. The following things have been done to help with the studying.

- Extra pages have been included next to the explanations to accommodate notes or examples that help to clarify what is written. By the time the course is over, the student will have written half of a math book.
- The exercises for each section of material are spread across 3 or more assignments to help students in their desire to retain information.
- The answers are included to help students get immediate feedback on their work.
- Problems are included to encourage the reading of and preparation for the next section.
- One answer on each assignment has been intentionally left incorrect in the first three chapters. Finding it should give the student a sense of independence and confidence.
- Summaries of the processes are included at the end of each chapter for quick reference.

This book is meant to be a supportive resource where teacher and student have frequent interaction, not as an independent study course. In particular, the success of this book hinges upon the following:

- Repetition at an appropriate frequency is essential to good education
- Students desire to understand the material
- A teacher is present to supply explanations and examples and answer questions
- Timely and accurate feedback is essential to learning
- Preparation before class aids in comprehension

Good Luck.

Math 100A
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Chapter 1:
ARITHMETIC

Overview

Arithmetic

Section 1.1

Facts

Everyone has to start somewhere, and that start, for you, is right here. When you first started learning math, you probably learned the names for numbers and then you started to add: 3apples + 7apples equals how many apples? Well 10, of course.

My guess is that you caught on to what you were doing and can now add m&m's, coconuts, gallons of water, money etc. From the beginning I am going to assume you know how to add in your head up to 15+15. If you don't, please make up some flash cards and get those in your brain. It is similar to learning the alphabet before learning to read. We need the addition facts to be available for instant recall.

Soon after addition was learned, I bet someone told you that there was a shortcut when you had to add some numbers up over and over. For example:

$$\underbrace{3+3+3+3+3+3+3}_{7} = 21$$

If you notice, there are seven 3's. 3, seven times, turns out to be 21, so we write it as 7x3 = 21.

One of the best coincidences of the world is that 7, three times, is also 21.

3x7 = 21

Such a switching works for any numbers we pick:

4x5 = 20 and 5x4 = 20

3x13 = 39 and 13x3 = 39

Since we will be using the multiplication facts almost as much as we will be using the addition facts, you need to also memorize the multiplication facts up to 15x15. Learn them well and you will be able to catch on to everything else quite nicely.

Section 1.1 Exercises

1.1

1. Make flash cards up to 15+15 and 15x15.
2. Memorize the addition and multiplication facts up to 15+15 and 15x15.
3. Fill out Addition/Subtraction Monster. Time yourself. Write the time it takes on the paper.
4. Fill out Multiplication Monster. Time yourself. Write the time on the paper.

Addition/Subtraction Monster

Name ___________________

12+13=	5+6=	5+10=	12-9=	5+9=	8+11=	5+11=	14-4=
6+6=	7+12=	15-8=	10+10=	10-7=	6+11=	6+12=	6+13=
7+7=	14-7=	7+9=	9+13=	6+14=	15-5=	11+11=	7-5=
12-4=	10+12=	8+10=	13-8=	5+5=	8+13=	5+12=	7+8=
9+9=	5+15=	9+11=	9+12=	15-6=	13-5=	9+15=	8+15=
6+7=	13-9=	8+12=	10+13=	10+14=	10+15 =	7+13=	11+13=
5+7=	11+12=	14-9=	11+14=	11+15=	8+9=	10-6=	8-7=
12+12=	6+10=	12+14=	8+8=	12-7=	12-8=	14+14=	12-6=
9-7=	13+14=	10-5=	7+14=	6+9=	13-7=	13-6=	9+10=
6+8=	14+15=	14-10=	12+15=	14-8=	8+14=	14-6=	10+11=
8-5=	15-11=	15-10=	15-9=	9-8=	7+10=	9+14=	13+15=
7+11=	5+14=	6+15=	15-7=	5+13=	7+15=	5+8=	7-6=
13+13=	8-6=	9-5=	9-6=	15-4=	15+15=	13-4=	14-5=

Assignment 1.1

Multiplication Monster

Name ________________

12x13=	5x6=	5x10=	12x9=	5x9=	8x11=	5x11=	14x4=
6x6=	7x12=	15x8=	10x10=	10x7=	6x11=	6x12=	6x13=
7x7=	14x7=	7x9=	9x13=	6x14=	15x5=	11x11=	7x5=
12x4=	10x12=	8x10=	13x8=	5x5=	8x13=	5x12=	7x8=
9x9=	5x15=	9x11=	9x12=	15x6=	13x5=	9x15=	8x15=
6x7=	13x9=	8x12=	10x13=	10x14=	10x15 =	7x13=	11x13=
5x7=	11x12=	14x9=	11x14=	11x15=	8x9=	10x6=	8x7=
12x12=	6x10=	12x14=	8x8=	12x7=	12x8=	14x14=	12x6=
9x7=	13x14=	10x5=	7x14=	6x9=	13x7=	13x6=	9x10=
6x8=	14x15=	14x10=	12x15=	14x8=	8x14=	14x6=	10x11=
8x5=	15x11=	15x10=	15x9=	9x8=	7x10=	9x14=	13x15=
7x11=	5x14=	6x15=	15x7=	5x13=	7x15=	5x8=	7x6=
13x13=	8x6=	9x5=	9x6=	15x4=	15x15=	13x4=	14x5=

Section 1.2

Rounding and Estimation

Now, you know that some problems may get long and tedious, so you can understand why some folks choose to **estimate** and **round** numbers. Rounding is the quickest, so we will tackle that first.

In rounding, we decide to not keep the exact number that someone gave us. For example:

If I have $528.37 in the bank, I might easily say that I have about $500. I have just rounded to the nearest hundred.

On the other hand, I might be a little more specific and say that I have about (still not exact) $530. I have just rounded to the nearest ten.

Example:

Round to the nearest hundredth:

538.4691

This number is right between 538.46 and 538.47

Which one is nearest? The 9 tells us that we are closer to

538.47

2nd Example:

Round to the nearest thousand:

783,299.4321

This number is right between 783,000 and 784,000

Which one is nearest? The 2 in the hundreds tells us that we are closer to :

783,000

Estimation

Once rounding is understood, it can be used as a great tool to make sure that we have not missed something major in our computations. If we have a problem like:

$$\begin{array}{r} 3{,}427{,}000 \\ \underline{\times \quad 87.3} \end{array}$$

We could see **about** where the answer is if we estimate first:
Round each number to the greatest value you can

$$\begin{array}{r} 3{,}000{,}000 \\ \underline{\times \quad 90} \end{array}$$

Voila! Our answer will be **around** 270,000,000

We should note that the **real** answer is:

299,177,100

but the estimation will let us know that we are in the right ball park. It ensures that our answer makes sense.

Estimation
1. Round to the highest value.
2.Do the easy problem.

Examples

Section 1.2 Exercises

1.1

1. Finish memorizing the addition and multiplication facts up to 15+15 and 15x15 until you know them all.

2. Fill out Addition/Subtraction Monster. Time yourself. Write the time it takes on the paper.

3. Fill out Multiplication Monster. Time yourself. Write the time on the paper.

1.2

Estimate the following.

4. 21 x 3250.07

5. 138.9 x 2892

6. 42 x 189

7. 369.456 ÷ 3.987

8. 58 x 39

9. 351 x 44

Answers:

1 I hope you have it done.
2. Check your cards.
3. Check your other cards.
4. 60,000
5. 300,000
6. 8,000
7. 100
8. 58,000
9. 16,000

Addition/Subtraction Monster

Name __________________

12+13=	5+6=	5+10=	12-9=	5+9=	8+11=	5+11=	14-4=
6+6=	7+12=	15-8=	10+10=	10-7=	6+11=	6+12=	6+13=
7+7=	14-7=	7+9=	9+13=	6+14=	15-5=	11+11=	7-5=
12-4=	10+12=	8+10=	13-8=	5+5=	8+13=	5+12=	7+8=
9+9=	5+15=	9+11=	9+12=	15-6=	13-5=	9+15=	8+15=
6+7=	13-9=	8+12=	10+13=	10+14=	10+15 =	7+13=	11+13=
5+7=	11+12=	14-9=	11+14=	11+15=	8+9=	10-6=	8-7=
12+12=	6+10=	12+14=	8+8=	12-7=	12-8=	14+14=	12-6=
9-7=	13+14=	10-5=	7+14=	6+9=	13-7=	13-6=	9+10=
6+8=	14+15=	14-10=	12+15=	14-8=	8+14=	14-6=	10+11=
8-5=	15-11=	15-10=	15-9=	9-8=	7+10=	9+14=	13+15=
7+11=	5+14=	6+15=	15-7=	5+13=	7+15=	5+8=	7-6=
13+13=	8-6=	9-5=	9-6=	15-4=	15+15=	13-4=	14-5=

Multiplication Monster

Name _________________

12x13=	5x6=	5x10=	12x9=	5x9=	8x11=	5x11=	14x4=
6x6=	7x12=	15x8=	10x10=	10x7=	6x11=	6x12=	6x13=
7x7=	14x7=	7x9=	9x13=	6x14=	15x5=	11x11=	7x5=
12x4=	10x12=	8x10=	13x8=	5x5=	8x13=	5x12=	7x8=
9x9=	5x15=	9x11=	9x12=	15x6=	13x5=	9x15=	8x15=
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7x11=	5x14=	6x15=	15x7=	5x13=	7x15=	5x8=	7x6=
13x13=	8x6=	9x5=	9x6=	15x4=	15x15=	13x4=	14x5=

Assignment 1.2

Section 1.3
Addition

So, you are able to add:

3penguins + 9penguins = 12 penguins

But have you thought about what would happen if you tried

3penguins + 9sheep = ?

If you are anything like me you have the desire to say something like, “Bzzzzzzz, thanks for playing” or, “Nice try, wise guy”, or if you are feeling quite clever, you might say, “12 animals”.

Aha! Your little cleverness has unearthed one of the most fundamental and powerful math principles that you will ever learn. You can’t add things that aren’t alike. You first had to find out how they were similar before adding them. Consider this a cardinal rule:

Add like things.

This principle will apply to all addition everywhere you see it, even within numbers themselves. To illustrate this we are going to break up a number to see what is really inside. Can you remember that

32,584

is really 3 ten thousands, 2 thousands, 5 hundreds, 8 tens, and 4 ones all together in one number? We could also write it like this:

32,584 = 3(10,000) + 2(1000) + 5(100) + 8(10) + 4(1)

Just to make sure you are clear on it, here is a big example:

Billions
Hundred Millions
Ten Millions
Millions
Hundred Thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones
Tenths
Hundredths
Thousandths
Ten Thousandths
Hundred Thousandths
Millionths

6,731,239,465.726409

Which expanded is:

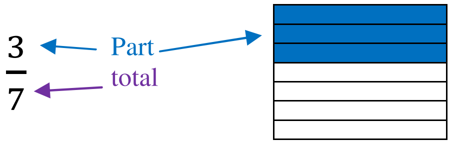
6(billion) + 7(100million) + 3(10million) + 1(million) + 2(100,000) + 3(10,000) + 9(1000) + 4(100) + 6(10) + 5(1) + 7(10^{th}) + 2(100^{th}) + 6(1000^{th}) + 4($10{,}000^{th}$) + 9(Millionths)

or could be written as:

6(billion) + 7(100million) + 3(10million) + 1(million) + 2(100,000) + 3(10,000) + 9(1000) + 4(100) + 6(10) + 5(1) + $\frac{7}{10}$ + $\frac{2}{100}$ + $\frac{6}{1000}$ + $\frac{4}{10{,}000}$ + $\frac{9}{1{,}000{,}000}$

It is important that you can see that the two ways of writing the number are exactly the same. Keep studying this example until you really can see that.

Notice how confining decimals can be. What if you cut a pie up in to 7 pieces? It isn't a perfect 10th or 100th or 1000th. What is great about fractions is that you can still write having three pieces of the pie as the number:



One may ask why in the world, we would break up a number like that. Though it may be a little tedious, it actually gives rise to every single one of the rules of addition.

We know that we **add like things:**

3apples + 2apples = 5apples
3(100) + 2(100) = 5(100)
3million + 2million = 5 million
3(thousandths) + 2(thousandths) = 5(thousandths)
$\frac{3}{100} + \frac{2}{100} = \frac{5}{100}$

So, when we have a problem:

$$\begin{array}{r} 23.27 \\ \underline{+13.024} \end{array}$$

We better add the (tens) to the (tens), the (hundredths) to the (hundredths) and the apples to the apples etc.

To make sure we add the right things together, we **line up the decimals.** Like this:

$$\begin{array}{r} 23.27 \\ \underline{+13.524} \end{array}$$

Which has as the answer 36.794

Now don't write it out but notice how nice it is that we have little problems like:
2(10) + 1(10) = 3(10)
3 + 3 = 6
$\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$
$\frac{7}{100} + \frac{2}{100} = \frac{9}{100}$
$\frac{0}{1000} + \frac{4}{1000} = \frac{4}{1000}$

Examples

Unfortunately not all of the problems are quite that clean. Try this one for example:

$$\begin{array}{r}378\\ \underline{+291}\end{array}$$

8 + 1 = 9 is fine,
And 3(100) + 2(100) = 5(100)
but 7(10) +9(10) = 16(10) – How in the world do you write that?

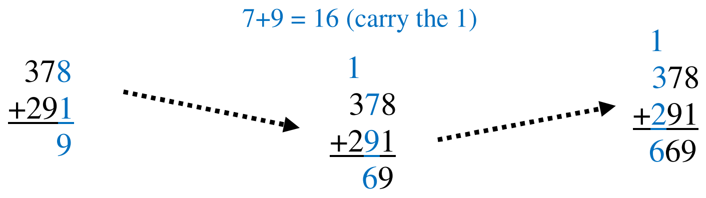
Regrouping

Let us digress here for a moment and think about a little money. Every time you get 10 pennies, you can cash them in for a dime. Every time you get 10 dimes, you can cash them in for a dollar. Every time you get 10 dollars, you can cash them in for a $10 bill. Every time you get 10 of those, you can cash it in for a $100. This idea of cashing in 10 items to get one bigger item is called **regrouping**, because you are putting those 10 items into a completely different group.

That last problem becomes
8 + 1 = 9 is fine,
7(10) +9(10) = 16(10) ***cash 10 of them in for 100*** = 1(100) and 6(10)
And 3(100) + 2(100) = 5(100)

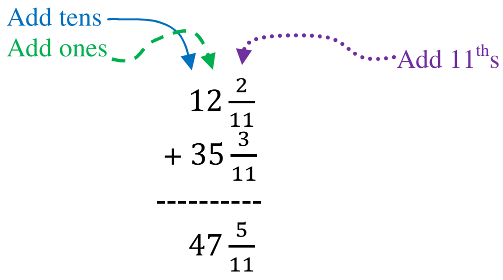
Total: 6(100) + 6(10) + 9(1)
669

Perhaps you have heard of "carrying" the one. Regrouping is where this idea comes from. Here is how it works with our example:



This idea of carrying, or regrouping helps us with all of our addition problems. Let's talk a minute about how fractions work.

Look at how boring decimals can be. Everything is in 10ths or 100ths or 1000ths. But with fractions you can have anything you want, such as a 3rd, or 7th, or 295th. Just make sure you know that 11(11ths) is 1 as well as $\frac{23}{23}=1$, and $\frac{7}{7}=1$. That will help us in our carrying. We still always add like things:



Examples

Let's again look at money to see how regrouping would work in fractions. Instead of working with dimes and pennies, let's try quarters or 4th's of a dollar. Notice that it doesn't take 10 of them before we have a dollar, it only takes 4.

Example: \$5 and 3 quarters added to \$7 and 2 quarters:

$$5\frac{3}{4} + 7\frac{2}{4} = 12\frac{5}{4}$$

We take 4 of the quarters and make a whole new dollar out of it. That's regrouping.

$$= 12 + 1 + \frac{1}{4}$$

$$= 13\frac{1}{4}$$

One last fraction example:

$$271\frac{9}{13} + 41\frac{7}{13} = 312\frac{16}{13}$$

We take 13 of the 13th's and make a whole new one out of it. That's regrouping.

Notice that we also regrouped when we added the tens. 4+7=11 so we had to carry the 1 to the next column.

$$= 312 + 1 + \frac{3}{13}$$

$$= 313\frac{3}{13}$$

Examples

Section 1.3 Exercises

1.1

1. **Fill out** Addition/Subtraction Monster. **Time** yourself. Write the time it takes on the paper.
2. **Fill out** Multiplication Monster. **Time** yourself. Write the time on the paper.

Estimate the following.

1.2

3. 28 x 3250.07 **4.** 158.9 x 7892 **5.** 82 x 189

6. Round 839.067 to the nearest hundred.
7. Round 839. 067 to the nearest hundredth.
8. Round 23,481,253.7 to the nearest hundred thousand.
9. Round 23,481,253.7 to the nearest ten thousand.

Write the numbers in expanded form.

Example:
35,421.78 = 3(10,000) + 5(1000) + 4(100) + 2(10) +1(1) +7(10th) +8(100th)

1.3

10. 4,368 **11.** 842.543 **12.** 8.9212

13. 567,420.1 **14.** 56,300.4 **15.** 57.476

16. 3,892,004 **17.** 26,349,700 **18.** .000754

19. 8.00671 **20.** 42,300.0078 **21.** 26,345

Add.

Example:

$$\begin{array}{r} 32.51 \\ \underline{+4.73} \end{array} \dashrightarrow \begin{array}{r} {}^{1} \\ 32.51 \\ \underline{+4.73} \\ 37.24 \end{array}$$

Example:

$$15\tfrac{8}{19} + 7\tfrac{13}{19} = \dashrightarrow \begin{array}{l} 15\tfrac{8}{19} + 7\tfrac{13}{19} = \\ 22\tfrac{21}{19} = \\ 22 + 1 + \tfrac{2}{19} = \\ 23\tfrac{2}{19} \end{array}$$

22. $\begin{array}{r} 234 \\ \underline{+57} \end{array}$ 23. $\begin{array}{r} 18.275 \\ \underline{+74.63} \end{array}$ 24. $4\tfrac{3}{7} + 2\tfrac{1}{7} =$

25. $\begin{array}{r} 4{,}386 \\ \underline{+5{,}119} \end{array}$ 26. $\begin{array}{r} 2.35 \\ \underline{+17.986} \end{array}$ 27. $\tfrac{2}{5} + \tfrac{2}{5} =$

28. $\begin{array}{r} 2,984 \\ \underline{+\ \ 151} \end{array}$

29. $\begin{array}{r} 15.346 \\ \underline{+4.92} \end{array}$

30. $5\frac{2}{11}+7\frac{10}{11}=$

31. $\frac{7}{13}+7\frac{4}{13}=$

32. $2\frac{10}{11}+935\frac{6}{11}=$

33. $\begin{array}{r} 58,349.062 \\ \underline{+5,432.27} \end{array}$

Find the final result.

Example:
Cost: $5,380
Tax: $1,055
Final Price:

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \ \ \\ 5,380 \\ \underline{+1,055} \\ 6,435 \end{array}$$

Final Price: $6,435

34. Cost:$32.50
Tax:$1.79
Final Price:

35. Temp:37° F
Change:8° warmer
Final:

36. Altitude:5,000 ft
Rise:3,200 ft
Final:

37. Cost: $264.15
Tax:$43.28
Final Price:

38. Temp: 73° F
Change: 8° warmer
Final:

39. Altitude: 1532m
Rise:1048m
Final:

40. Preparation.

Find the answer to these problems.

5 – 7 =

10 – 23 =

8 trees – 15 trees =

7° F - 13° F =

Addition/Subtraction Monster 2

9-6=	12-4=	5+10=	6+15=	15-5=	8+11=	12-9=	14-4=
6+6=	9-7=	15-8=	10+10=	10-7=	6+11=	13-7=	5+8=
7+7=	7+12=	15-10=	9+13=	6+14=	12+13=	7-5=	13+15=
5+11=	10+12=	8+10=	15-7=	14-7=	8+13=	5+12=	7+8=
9+9=	5+15=	9+11=	9+12=	6+13=	5+5=	9+15=	8+15=
6+7=	11+15=	8+12=	13-5=	10+14=	10+15=	7+13=	11+13=
5+7=	11+12=	11+11=	11+14=	13-8=	8+9=	10-6=	5+9=
12+12=	14-9=	12+14=	8+8=	12-7=	10+13=	14+14=	12-6=
15+15=	13+14=	10-5=	7+14=	12-8=	6+8=	13-6=	9+10=
5+6=	14+15=	6+10=	12+15=	14-8=	8+14=	14-6=	10+11=
8-5=	15-11=	13-9=	15-9=	6+9=	7+10=	9+14=	7-6=
7+11=	5+14=	15-6=	6+12=	14-10=	7+15=	9-8=	7+9=
13+13=	8-6=	9-5=	5+13=	15-4=	8-7=	13-4=	14-5=

Multiplication Monster 2

9x6=	12x4=	5x10=	6x15=	15x5=	8x11=	12x9=	14x4=
6x6=	9x7=	15x8=	10x10=	10x7=	6x11=	13x7=	5x8=
7x7=	7x12=	15x10=	9x13=	6x14=	12x13=	7x5=	13x15=
5x11=	10x12=	8x10=	15x7=	14x7=	8x13=	5x12=	7x8=
9x9=	5x15=	9x11=	9x12=	6x13=	5x5=	9x15=	8x15=
6x7=	11x15=	8x12=	13x5=	10x14=	10x15=	7x13=	11x13=
5x7=	11x12=	11x11=	11x14=	13x8=	8x9=	10x6=	5x9=
12x12=	14x9=	12x14=	8x8=	12x7=	10x13=	14x14=	12x6=
15x15=	13x14=	10x5=	7x14=	12x8=	6x8=	13x6=	9x10=
5x6=	14x15=	6x10=	12x15=	14x8=	8x14=	14x6=	10x11=
8x5=	15x11=	13x9=	15x9=	6x9=	7x10=	9x14=	7x6=
7x11=	5x14=	15x6=	6x12=	14x10=	7x15=	9x8=	7x9=
13x13=	8x6=	9x5=	5x13=	15x4=	8x7=	13x4=	14x5=

Answers:

1. Check your flash cards

2. Check your flash cards

3. 90,000

4. 1,600,000

5. 16,000

6. 800

7. 839.07

8. 23,500,000

9. 23,480,000

10. 4(1000)+3(100)+6(10)+8(1)

11. 8(100)+4(10)+2(1)+5(10^{th})+4(100^{th})+3(1000^{th})

12. 8(1)+9(10^{th})+2(100^{th})+1(1000^{th})+2($10{,}000^{th}$)

13. 5(100,000)+6(10,000)+7(1000)+4(100)+2(10)+1(10^{th})

14. 5(10,000)+6(1000)+3(100)+4(10^{th})

15. 5(10)+7(1)+4(10^{th})+7(100^{th})+6(1000^{th})

16. 3(million)+8(100,000)+9(10,000)+2(1000)+4(1)

17. 2(10 million)+6(million)+ 3(100,000)+4(10,000)+9(1000)+7(100)

18. 7($10{,}000^{th}$)+5($100{,}000^{th}$)+4(millionths)

19. 8(1)+6(1000^{th})+7($10{,}000^{th}$)+1($100{,}000^{th}$)

20. 4(10,000)+2(1000)+3(100)+7(1000^{th})+8($10{,}000^{th}$)

21. 2(10,000)+6(1000)+3(100)+4(10)+5(1)

22. 291

23. 92.905

24. $6\frac{4}{7}$

25. 9,505

26. 20.336

27. $\frac{4}{5}$

28. 3,135

29. 20.266

30. $13\frac{1}{11}$

31. $7\frac{11}{13}$

32. $938\frac{5}{11}$

33. 63781.332

34. $34.29

35. 45° F

36. 8200ft

37. $820.43

38. 81° F

39. 2580 m

40. Let’s talk in class about it.

Section 1.4

Subtraction

Naturally following addition is subtraction. Instead of adding things together, we are starting with some and taking away from it. Luckily, the same rule applies.

We always **subtract like things.** Just like in addition:

7apples – 3apples = 4 apples

8(1000) – 2(1000) = 6(1000)

9(millionths) – 6(millionths) = 3(millionths)

$$\frac{5}{100} - \frac{2}{100} = \frac{3}{100}$$

$$\frac{9}{23} - \frac{7}{23} = \frac{2}{23}$$

Regrouping is just as necessary here as it was with addition. The only difference is that we regroup backwards. This example is a good beginning:

$$\begin{array}{r} 2378 \\ \underline{-1095} \end{array}$$

Our problem lies in the fact that we can't really take 9 tens away from 7 tens. Here is where the concept of regrouping is fantastic. If you look to the left of the 2, there are 3 hundreds just hanging out. One of them is the same as 10 tens. This kind of "un-regrouping" is usually called borrowing.

Here is how the problem looks:

We "borrowed" 10 tens from the 300 so now we have 2 hundreds and 17 tens. Subtract as normal.

$$\begin{array}{r} 2378 \\ \underline{-1095} \\ 3 \end{array} \dashrightarrow \begin{array}{r} 2\,\cancel{3}^{2}\,{}^{1}7\;\,8 \\ \underline{-1\;\,0\;\;9\;\,5} \\ 3 \end{array} \dashrightarrow \begin{array}{r} 2\,\cancel{3}^{2}\,{}^{1}7\;\,8 \\ \underline{-1\;\,0\;\;9\;\,5} \\ 1\;\,2\;\,8\;\,3 \end{array}$$

An example with fractions:

$$\begin{array}{r} 4\frac{2}{11} \\ -1\frac{8}{11} \end{array} \dashrightarrow \begin{array}{r} 3\frac{13}{11} \\ \underline{-1\frac{8}{11}} \\ 2\frac{5}{11} \end{array}$$

Look, we "borrowed $\frac{11}{11}$ from the 4 so that we could subtract as normal.

Subtraction inevitably leads to the question of, “What if I take away more than I have?” It is a question worthy of answering. What if you have $35 in the bank and you write a check for $45? What if the temperature is 15° and then drops 20°? We need to discuss this a bit.

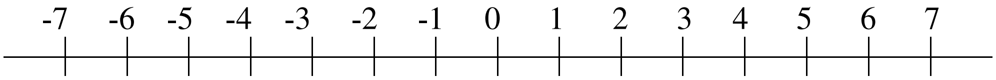
Taking these examples gives us a really good idea of what is really going on. With the money example, where we start with $35 and write a check for $45, we bounce the check. The bank sends us a statement that says we have $-10 in the account. Naturally the question arises as to whether that can actually happen.

Sometimes I have wanted to take such a bank statement to the bank and show them the statement and demand to see the $-10 which are sitting in my account. Go ahead. See if you can count -10 anything . . . penguins, apples, dollars or degree. Negative numbers simply don’t exist; we can’t really count anything with them, but the bank, and you, know that we really do use them. What does the -$10 really mean? It means that ***we owe*** the bank $10. The **negative** simply means that the $10 is going **the other direction**. We owe the bank $10.

Negative sign = opposite direction

Same thing with the temperature: If we start at 15° and lose 20°, we will have passed right by 0° and end up at 5 below zero or -5°. Again, the **negative sign** means that we are going 5° past 0° **in the opposite direction.**

From this concept of “other direction” we make a graph that expresses how the numbers look:



With this diagram it is easy to see:

Addition goes this way →

← Subtraction goes that way

And a **negative sign means other direction** ↩

Examples:

7 – 11 = -4
Start at 7, go left 11, past 0, to -4.

-3 – 4 = -7
Start at -3, go left 4 to -7

-2 – (-7) = 5
Start at -2, go left (switch direction , go right) 7 up to positive 5.

We start to see a couple of things that help us in subtraction. The first idea is that of a **tug-o-war**. Positives and negatives fight against each other in such a way that the stronger one wins. In the first example, you have a positive 7 pulling against a -11. The 11 is bigger so it beats the 7. Thus the answer is -4.

Examples

Example of the tug-o-war:

-18 + 23 = 5 since the 23 is larger, the answer ends up on the positive side of zero.

From the last example we get a quick and snappy way of subtracting negatives. Since we switch direction on the negative, it ends up being the same as adding:

3 – (-4) =	-5 – (-7) =	7 – (-18) =
3 + 4 = 7	-5 + 7 = 2	7 + 18 = 25

One of the most amazing things about subtraction is what happens when you do it backwards. For example:

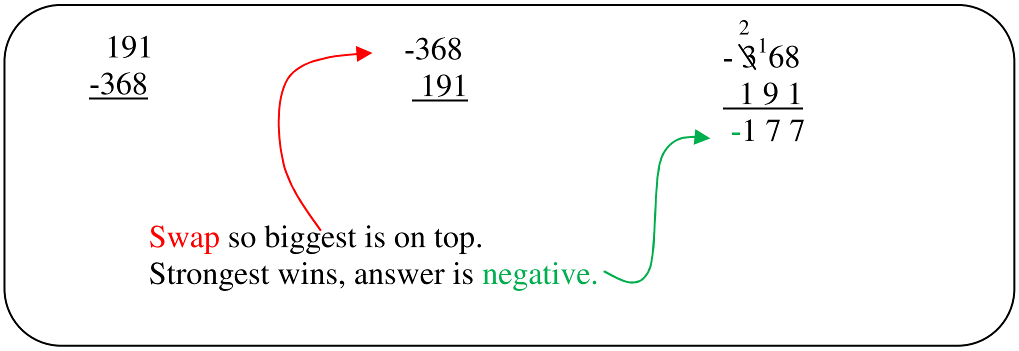
5 – 2 = 3	2 – 5 = -3
11 – 7 = 4	7 – 11 = -4

Which fact allows us to have a specific way of subtracting that will work for all types of subtraction problems. For example:

191 – 368 = ?

We don't know yet, but we do know that it will be the exact opposite of 368 – 191.

So, just make sure to **put the stronger number on top.** I like to keep the sign with the stronger number, so I know who wins the tug 'o war.



Swap so biggest is on top.
Strongest wins, answer is negative.

One more thing that illustrates the concept and use of negatives: sometimes in practical application, there are words that we use instead of a negative sign. They include:

Loss, fall, lower, below zero, discount, negative, deep, sale, less than, decrease

All of these denote things that will be a negative. Watch for them.

In summary, all addition and subtraction problems with decimals can be can be done by following four simple steps:

Addition/Subtraction of Decimals:

1. Set it up
   1. Line up decimals
   2. Subtract with bigger on top
2. Add/Subtract Columns
3. Carry or borrow by 10's
4. Stronger number wins. (+ or -)

Section 1.4 Exercises

1.1

1. Fill out Addition/Subtraction Monster. Time yourself. Write the time it takes on the paper.
2. Fill out Multiplication Monster. Time yourself. Write the time on the paper.

Estimate the following.

1.2

3. 528 x 4250.045

4. 758.9 x 37,892

5. Round 85,434.967 to the nearest ten.

6. Round 839. 067 to the nearest tenth.

Write the numbers in expanded form.

1.3

7. 200.0786

8. 13,205

9. 5.37

Add.

10. $\begin{array}{r} 36{,}451 \\ \underline{+\ 2{,}197} \end{array}$

11. $\begin{array}{r} 143.29 \\ \underline{+\ .923} \end{array}$

12. $7\frac{13}{23} + 15\frac{15}{23} =$

13. $5\frac{11}{75} + 19\frac{73}{75} =$

14. $89\frac{10}{11} + 273\frac{7}{11} =$

15. $\begin{array}{r} 5{,}834{,}906.2 \\ \underline{+\ \ 54.3227} \end{array}$

Subtract.

1.4

Example:
$-8 - 3 = \xrightarrow{\quad} -8 - 3 = -11$

Example:
$4.3 - 15.21 = \xrightarrow{\quad} \begin{array}{r} 1\overset{4}{\cancel{5}}.\,^{1}21 \\ \underline{-\ 4.\ \ 3\ \,} \\ -10.\ 9\ 1 \end{array}$

16. 7- (-2) =

17. -7 – 2 =

18. -13 –(-10)

19. -18 + 5 =

20. 10 – 57 =

21. -14 – 8 =

22. $\begin{array}{r} 234 \\ \underline{-57} \end{array}$

23. $\begin{array}{r} 19.275 \\ \underline{-74.63} \end{array}$

24. $4\frac{3}{7} - 2\frac{1}{7} =$

25. $\begin{array}{r} 4{,}386 \\ \underline{-5{,}119} \end{array}$

26. $\begin{array}{r} 2.35 \\ \underline{-17.986} \end{array}$

27. $\frac{4}{5} - \frac{2}{5} =$

28. $\begin{array}{r} 2{,}984 \\ \underline{-\ \ 151} \end{array}$

29. $5\frac{7}{19} - 12\frac{2}{19}$

30. $9\frac{2}{11} - 7\frac{10}{11} =$

Find the final result.

Example:
Cost: $5,380
Discount: $1,055
Final Price:

$$\begin{array}{r} 5{,}38^{1}0 \\ \underline{-1{,}05\ 5} \\ 4{,}32\ 5 \end{array}$$

Final Price: $4,325

31. Cost:$32.50
Discount:$1.79
Final Price:

32. Temp:67° F
Change:18° warmer
Final:

33. Altitude: 7,380 ft
Fall: 3,200 ft
Final:

34. Cost:$32.50
Tax:$2.08
Final Price:

35. Temp: 17° C
Change: 28° colder
Final:

36. Altitude:300 m
Rise:7,250 m
Final:

37. Preparation.

Find the answer to this problem. Are there other possibilities?

$$5 + 3 \times 7 - 4 =$$

Addition/Subtraction Monster 2

9-6=	12-4=	5+10=	6+15=	15-5=	8+11=	12-9=	14-4=
6+6=	9-7=	15-8=	10+10=	10-7=	6+11=	13-7=	5+8=
7+7=	7+12=	15-10=	9+13=	6+14=	12+13=	7-5==	13+15=
5+11=	10+12=	8+10=	15-7=	14-7=	8+13=	5+12=	7+8=
9+9=	5+15=	9+11=	9+12=	6+13=	5+5=	9+15=	8+15=
6+7=	11+15=	8+12=	13-5=	10+14=	10+15 =	7+13=	11+13=
5+7=	11+12=	11+11	11+14=	13-8=	8+9=	10-6=	5+9=
12+12=	14-9=	12+14=	8+8=	12-7=	10+13=	14+14=	12-6=
15+15=	13+14=	10-5=	7+14=	12-8=	6+8=	13-6=	9+10=
5+6=	14+15=	6+10=	12+15=	14-8=	8+14=	14-6=	10+11=
8-5=	15-11=	13-9=	15-9=	6+9=	7+10=	9+14=	7-6=
7+11=	5+14=	15-6=	6+12=	14-10=	7+15=	9-8=	7+9=
13+13=	8-6=	9-5=	5+13=	15-4=	8-7=	13-4=	14-5=

Assignment 1.4

Multiplication Monster 2

9x6=	12x4=	5x10=	6x15=	15x5=	8x11=	12x9=	14x4=
6x6=	9x7=	15x8=	10x10=	10x7=	6x11=	13x7=	5x8=
7x7=	7x12=	15x10=	9x13=	6x14=	12x13=	7x5==	13x15=
5x11=	10x12=	8x10=	15x7=	14x7=	8x13=	5x12=	7x8=
9x9=	5x15=	9x11=	9x12=	6x13=	5x5=	9x15=	8x15=
6x7=	11x15=	8x12=	13x5=	10x14=	10x15	7x13=	11x13=
5x7=	11x12=	11x11	11x14=	13x8=	8x9=	10x6=	5x9=
12x12=	14x9=	12x14=	8x8=	12x7=	10x13=	14x14=	12x6=
15x15=	13x14=	10x5=	7x14=	12x8=	6x8=	13x6=	9x10=
5x6=	14x15=	6x10=	12x15=	14x8=	8x14=	14x6=	10x11=
8x5=	15x11=	13x9=	15x9=	6x9=	7x10=	9x14=	7x6=
7x11=	5x14=	15x6=	6x12=	14x10=	7x15=	9x8=	7x9=
13x13=	8x6=	9x5=	5x13=	15x4=	8x7=	13x4=	14x5=

Assignment 1.4

Answers:

1. Check your flash cards
2. Check your flash cards
3. 2,000,000
4. 32,000,000
5. 85,430
6. 839.1
7. 2(100)+7(100th)+8(1000th)+6(10,000th)
8. 1(10,000)+3(1000)+2(100)+5(1)
9. 5(1)+3(10th)+7(100th)
10. 38,648
11. 144.213
12. $23\frac{5}{23}$
13. $25\frac{9}{75}$ or $25\frac{3}{25}$
14. $363\frac{6}{11}$
15. 5,834,960.5227
16. 9
17. -9
18. -3
19. -13
20. -47
21. -22
22. 177
23. -55.355
24. $2\frac{2}{7}$
25. -894
26. -15.636
27. $\frac{2}{5}$
28. 2833
29. $-6\frac{14}{19}$
30. $1\frac{3}{11}$
31. $30.71
32. 85° F
33. 4180 ft
34. $34.58
35. -11° C
36. 7550 m
37. Talk about in class

Section 1.5

Multiplication

Multiplication: as we have already seen, multiplication is just addition repeated a number of times. Let's see how this works when we are multiplying things together.

Example:

30x20 =

$$\underbrace{30+30+30+30+30+30+30+30+30+30+30+30+30+30+30+30+30+30+30+30}_{20}$$

= 600.

Hmmmmm. This raises some questions. We know that we had 3(10) x 2(10), but we ended up with 6(hundreds). Let's see another one:

20x400

$$\underbrace{400+400+400+400+400+400+400+400+400+400+400+400+400+400+400+400+400+400+400+400}_{20}$$

= 8000

Hmmmmmm. We had 2(10) x 4(100) and ended up with 8(thousands). It appears that we don't need like things in order to do multiplication, and we certainly don't end up with the same "things" we started with. But there does seem to be a pattern:

1. We multiply the numbers.
2. We multiply the "things" of the numbers.

2(10) x 4(100)=
8(10x100) =
8(1000) = 8000

Luckily tens, hundreds and thousands are easy to multiply. All you do is count up the zeros.

4 zeros and 3 zeros total 7 zeros

Example: 50,000x7,000 = 350,000,000

Note also in this example that we actually carried the three to the next column. We do that any time we need to.

These same ideas work in decimals as well:

2 decimal places and
4 decimal places
total 6 decimal places

.02x.0004 =
$2(100^{th})x4(10,000^{th})$ =
$8(1,000,000^{th})$ =
.000008

Notice here that multiplying the "things" means that you just have to count up the decimal places.

And of course these ideas carry over very well into fractions to:

$3(7^{th})x2(7^{th}) = 6(49^{th})$

Or

$$\frac{3}{7} \times \frac{2}{7} = \frac{6}{49}$$

Notice here that multiplying the "things" means that you **multiply the denominators of the fractions** as well as the numerators.

Here is another example just to make sure we have this down:

$$\frac{4}{5} \times \frac{3}{11} = \frac{12}{55}$$

That's it.

Here is the summary of how:

Multiplication of Fractions

1. No common denominator. (Yippy!)
2. Multiply numerators.
3. Multiply denominators.
4. Simplify

Distributive property:

One of the most amazing features of our numbers is something called the distributive property. It says that there are a couple of different ways to do a problem such as:

3 x (5 + 4)

This particular problem isn't tough, but there are two ways to do it:

Distributive property

3 x (5 + 4) =
3 x 9 =
27

3 x (5 + 4) =
3x5 + 3x4 =
15 + 12 =
27

We distributed or "jumped" the 3 into the 4 and the 5.

Whoop-dee-doo, you say. Why in the world would we do something like that? It seems to just have made the problem more difficult. Well, it comes in really handy in certain situations, like:

5x37 = or 8x42 =

I don't know about you, but that answer doesn't just pop out at me, but, I do know that I could change it, and break it up into two really nice parts:

5x37 =
5 x (30 + 7) =
5x30 + 5x7 =
150 + 35 =
185

or

8x42 =
8 x (40 + 2) =
8x40 + 8x2 =
320 + 16 =
336

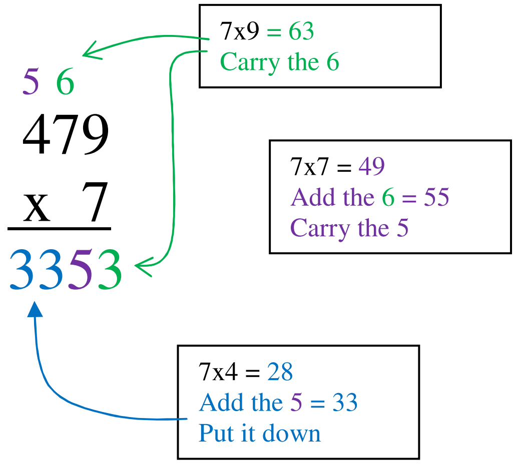
Man, that is slick! You could probably do a ton of those in your head! Not only does it allow you to break up math into easy chunks, but it also forms the basis for the steps, or algorithms, you use to do bigger multiplication. Take a look at:

3,692
x 8

We can do it just a piece at a time. Here we go in really slow motion:

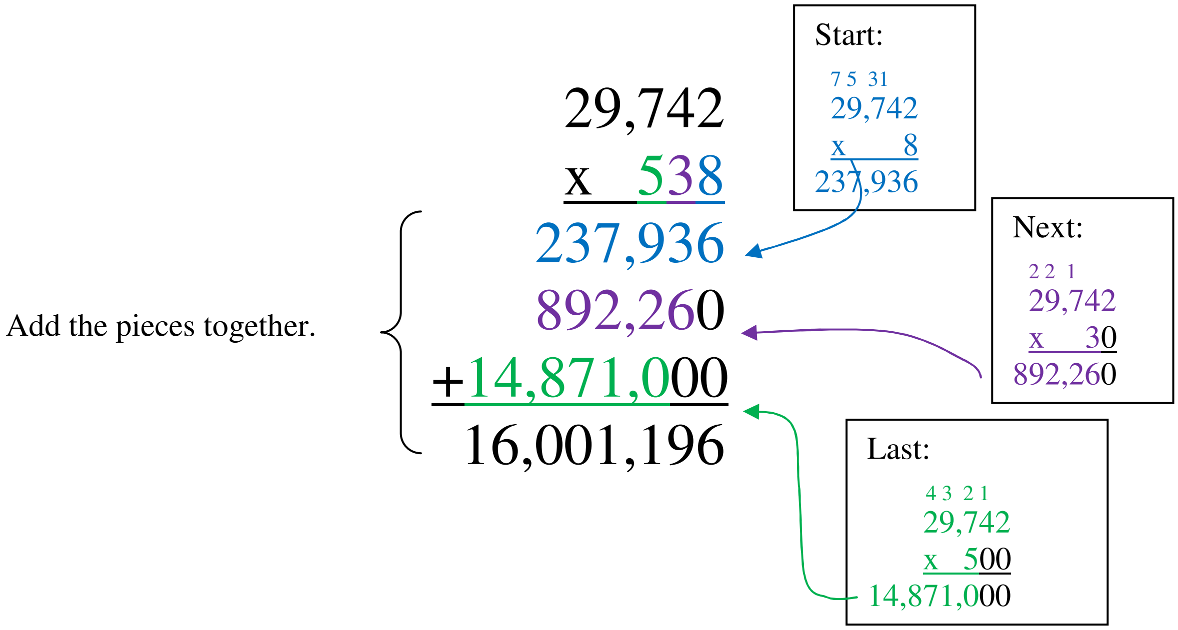
1 3, 6 9 2 x 8 6	First piece: 8x2 = 16 Which is 6(ones) and 1(ten). We carry the 1.
7 1 3, 6 9 2 x 8 3 6	Next piece: 8x90 = 720 Don't forget that we carried 1 extra 10. Add it on to get 730. Place the 3 down and carry the 7 (hundred)
5 7 1 3, 6 9 2 x 8 5 3 6	Next piece: 8x600 = 4800 Don't forget that we carried the 7 (hundred) Add it on to get 5500. Place the 5 down and carry the other 5
5 7 1 3, 6 9 2 x 8 29,5 3 6	Next piece: 8x3,000 = 24,000 Don't forget to add the extra 5,000 Add it to get 29,000 Finshed.

Notice how, when you lay everything down in its appropriate column (i.e. hundreds go in the hundreds slot etc.) and carry everything appropriately, you can just work from left to right and even go on indefinitely. Here is another example:



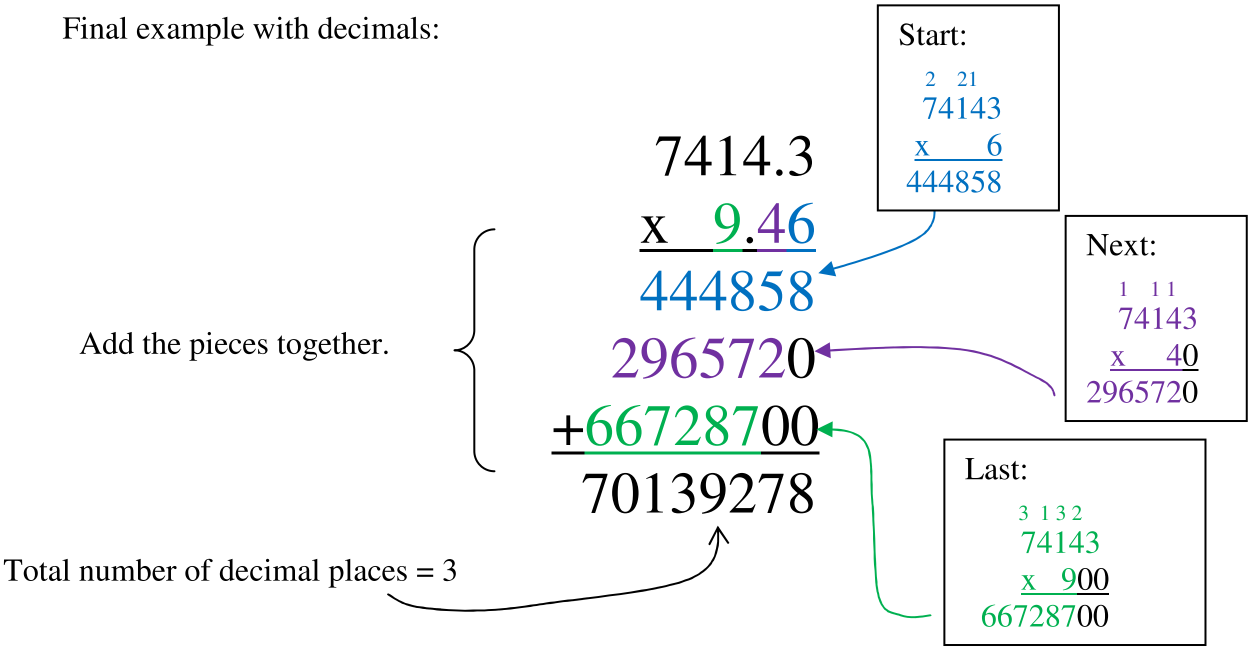
Examples

Once you have this multiplication down, even larger problems can be broken up into chunks as well. Made up of a lot of little pieces, here is a huge example:



If you noticed, I left some of the zeros in the example in black. Some find it easier to put the zeros down to make sure that all of the numbers end up in the right slot.

Final example with decimals:



The only thing left is to count the number of decimal places. We have one in the first number and two in the second. Final answer: 70139.278

Examples

Summary:

Multiplication of Decimals

1. Multiply each place value
2. Carry by 10’s
3. Add
4. Right size.
    1. Add up zeros or decimals
    2. Negatives

Section 1.5 Exercises

Estimate.

1.2

1. 5,670,800 x 39

2. .0003467 x .000561

3. Round 8.95647 to the nearest thousandth.

Add.

1.3

4.
$$\begin{array}{r} 56{,}757 \\ \underline{+\ \ 2{,}377} \end{array}$$

5.
$$\begin{array}{r} 443.79 \\ \underline{+\ \ 5.23} \end{array}$$

6. $12\frac{22}{23}+19\frac{7}{23}=$

7. $\frac{15}{71}+19\frac{5}{71}=$

8. $4\frac{1}{3}+5\frac{2}{3}=$

9.
$$\begin{array}{r} 25.371 \\ \underline{+\ \ 9.06} \end{array}$$

Subtract.

1.4

10. 5 – (-18) =

11. -3 +15 =

12. -12 – 37 =

13.
$$\begin{array}{r} 235 \\ \underline{-7{,}121} \end{array}$$

14.
$$\begin{array}{r} 34.35 \\ \underline{-17.986} \end{array}$$

15. $7\frac{4}{5}-10\frac{2}{5}=$

16.
$$\begin{array}{r} 2{,}357 \\ \underline{-\ \ \ 51} \end{array}$$

17. $5\frac{7}{19}-3\frac{4}{19}$

18. $9\frac{2}{11}-17\frac{10}{11}=$

Find the final result.

19. Cost:$252
Discount:$41.79
Final Price:

20. Temp:-28° F
Change:18.7° warmer
Final:

21. Altitude: 300 m
Fall: 558 m
Final:

22. Cost:$587
Tax:$39.08
Final Price:

23. Temp: 17° C
Change: 28.4° colder
Final:

24. Altitude:300 m
Rise:8,489 m
Final:

Multiply.

1.5

Example:
236.4
x .92

236.4
x .92
4728
+212760
217.488
3decimal places

Example:

$\frac{7}{5}\times\frac{9}{13}=$ → $\frac{7}{5}\times\frac{9}{13}=\frac{63}{65}$

25. 156x8 =

26. $\frac{5}{14}\times\frac{3}{2}=$

27. 2,386
x 7

28. 2.35x9 =

29. 536x23 =

30. $\frac{5}{9}\times\frac{11}{8}=$

31. 2.385
x 5.2

32. $\frac{5}{7}\times\frac{9}{11}=$

33. 172x244 =

34. 781
x 43

35. .0055x55 =

36. $\frac{2}{13}\times 5=$

Find the final result.

Example:
Cost: $8.35
Quantity: 19
Total:

8.35
x 19
7515
+8350
158.65
2 decimal places

Total: $158.65

37. Cost: $5.37
Quantity: 7
Total:

38. Length: $\frac{5}{16}$ inch
Quantity: 3
Total length:

39. Cost: $26.50
Quantity: 14
Total:

40. Length: 17.2 ft
Quantity: 5
Total length:

41. Preparation.

Find.

15x14 = 13x11 = 12x7 = 8x9 =

Answers:

1. 240,000,000
2. .00000018
3. 8.956
4. 59,134
5. 449.02
6. $32\frac{6}{23}$
7. $19\frac{20}{71}$
8. 10
9. 34.431
10. 23
11. 12
12. -49
13. -6,886
14. 16.364
15. $-2\frac{3}{5}$
16. 2,306
17. $2\frac{3}{19}$
18. $-8\frac{8}{11}$
19. $210.21
20. -9.3° F
21. -258 m or 258 below sea level
22. $626.08
23. -11.4° C
24. 8,789 m
25. 1248
26. $\frac{15}{28}$
27. 16,702
28. 21.15
29. 12,493
30. $\frac{55}{72}$
31. 12.402
32. $\frac{45}{77}$
33. 41,968
34. 33,583
35. .3025
36. $\frac{10}{13}$
37. $37.59
38. $\frac{15}{16}$
39. $371
40. 86 ft
41. Let's talk about it.

Mid-Chapter Review

Write the numbers in expanded form.

1.3

1. 50,000.89

2. 20.304

3. 19,300

Add.

4. $\begin{array}{r} 389 \\ \underline{+\ 2{,}477} \end{array}$

5. $\begin{array}{r} 483.74 \\ \underline{+\ 9.93} \end{array}$

6. $16\frac{8}{23}+35\frac{17}{23}=$

7. $3\frac{55}{71}+19\frac{5}{71}=$

8. $\frac{4}{5}+\frac{3}{5}=$

9. $\begin{array}{r} 13.58 \\ \underline{+\ .706} \end{array}$

1.4

Subtract.

10. -5 – (-18) =

11. -3 + (-15) =

12. 12 – 37 =

13. $\begin{array}{r} 1{,}235 \\ \underline{-7{,}121} \end{array}$

14. $\begin{array}{r} 234.35 \\ \underline{-17.986} \end{array}$

15. $17\frac{4}{5}-10\frac{3}{5}=$

16. $\begin{array}{r} 2{,}357 \\ \underline{-7{,}851} \end{array}$

17. $5\frac{7}{19}-3\frac{9}{19}$

18. $9\frac{10}{11}-17\frac{2}{11}=$

Find the final result.

19. Cost:$248
Discount:$47.79
Final Price:

20. Temp:-15° F
Change:18.7° warmer
Final:

21. Altitude: 1300 m
Fall: 558 m
Final:

22. Cost:$687
Tax:$45.74
Final Price:

23. Temp: 52° C
Change: 28.4° colder
Final:

24. Altitude:3100 m
Rise:25.3 m
Final:

Multiply.

1.5

25. 769x5 =

26. $\frac{5}{14}\times\frac{3}{7}=$

27. $\begin{array}{r}4{,}827\\ \underline{\text{x}\quad 5}\end{array}$

28. 3.8x7.2 =

29. 219x63 =

30. $-\frac{7}{9}\times(-\frac{11}{13})=$

31. $\begin{array}{r}5.985\\ \underline{\text{x}\ \ 7.2}\end{array}$

32. $-\frac{5}{7}\times\frac{3}{11}=$

33. 381x237 =

34. $\begin{array}{r}741\\ \underline{\text{x}\ 57}\end{array}$

35. .36x.55 =

36. $\frac{3}{19}\times 6=$

Find the final result.

37. Cost: $8.96
Quantity: 4
Total:

38. Length: $\frac{1}{8}$ inch
Quantity: 7
Total length:

39. Cost: $27.80
Quantity: 19
Total:

40. Length: 7.6 m
Quantity: 8
Total length:

41. Preparation.

5x5x5 =

2x2x2x2x2x2=

3x3x3x3=

Answers:

1. 5(10,000)+8(10^{th})+9(100^{th})

2. 2(10)+3(10^{th})+4(1000^{th})

3. 1(10,000)+9(1,000)+3(100)

4. 2,866

5. 493.67

6. $52\frac{2}{23}$

7. $22\frac{60}{71}$

8. $1\frac{2}{5}$ or $\frac{7}{5}$

9. 14.286

10. 13

11. -18

12. -25

13. -5,886

14. 216.364

15. $7\frac{1}{5}$

16. -5,494

17. $1\frac{17}{19}$

18. $-7\frac{3}{11}$

19. $200.21

20. 3.7° F

21. 742 m

22. $732.74

23. 23.6° C

24. 3125.3 m

25. 3,845

26. $\frac{17}{21}$

27. 24,135

28. 27.36

29. 13,797

30. $\frac{77}{117}$

31. 43.092

32. $-\frac{15}{77}$

33. 90,297

34. 42,237

35. .198

36. $\frac{18}{19}$

37. $35.84

38. $\frac{7}{8}$ inch

39. $528.20

40. 60.8 m

41. Let’s talk about it in class.

Section 1.6

Exponents

While we are on multiplication, did you know that there is some short hand? Remember when we started multiplication we did:

$\underbrace{6+6+6+6+6+6+6+6+6}_{9} = 54$ but we did it a bit shorter

$9x6 = 54$

There is a way to write multiplication in shorthand if you do the same thing over and over again:

$$\underbrace{2x2x2x2x2x2x2}_{7} = 128$$

For the shorthand we write $2^7 = 128$.

That little 7 means the number of times that we multiply 2 by itself and is call and **exponent** sometimes we call it a **power**. Here are a couple more examples:

$5^3 = 125$ $7^2 = 49$ $2^4 = 16$

Pretty slick. You won’t have to memorize them . . . yet, but you should be familiar enough with them to be able to find them.

Some of the easiest to find are the powers of 10. Try these:

$10^4 = 10,000$ $10^8 = 100,000,000$ $10^3 = 1,000$

Section 1.6 Exercises

Add.

1.3

1. $9\frac{25}{43}+15\frac{32}{43}=$

2. $\frac{2}{7}+\frac{6}{7}=$

3. $\begin{array}{r} 9.78 \\ \underline{+\ .365} \end{array}$

Subtract.

1.4

4. -9 – (-2) =

5. -5 + (-1) =

6. 45 - 38 =

7. $\begin{array}{r} 4{,}511 \\ \underline{-3{,}849} \end{array}$

8. $\begin{array}{r} 34.006 \\ \underline{-57.984} \end{array}$

9. $10\frac{9}{13}-17\frac{3}{13}=$

Find the final result.

10. Cost:$589.27
Discount:$37.81
Final Price:

11. Temp: -23° F
Change:38.7° warmer
Final:

12. Altitude: 589 m
Fall: 1300 m
Final:

Multiply.

1.5

13. .046 x 67 =

14. $\frac{5}{9}\times\frac{8}{7}=$

15. $\begin{array}{r} 4{,}891{,}000 \\ \underline{\text{x}\qquad\quad 8} \end{array}$

16. $-\frac{5}{19}\times(-3)=$

17. $-\frac{15}{13}\times\frac{9}{11}=$

18. 291x862 =

Find the final result.

19. Cost: $48.96
Quantity: 4
Total:

20. Length: $\frac{3}{32}$ inch
Quantity: 7
Total length:

Find.

1.6

Example:
3^4 =

3^4 =
3x3x3x3 =
9 x3x3 =
27 x3 =
81

21. 5^2=

22. 2^3=

23. 4^2=

24. 7^2=
25. 8^3=
26. 10^2=
27. 4^4=
28. 3^5=
29. 2^6=
30. 9^2=
31. 10^4=
32. 5^3=
33. 10^5=
34. 7^3=
35. 2^5=

36. Preparation. Find:
10^{23}=
10^7=
10^6=
10^5=
10^4=
10^3=
10^2=
10^1=

What do you think these are?
10^0=
10^{-1}=
10^{-2}=

Answers:

1. $25\frac{14}{43}$
2. $1\frac{1}{7}$ or $\frac{8}{7}$
3. 10.145
4. -7
5. -6
6. 7
7. 662
8. -23.978
9. $-6\frac{7}{13}$
10. $551.46
11. 15.7°F
12. -711 m or 711 m below sea level
13. 3.082
14. $\frac{40}{63}$
15. 39,128,000
16. $\frac{15}{19}$
17. $-\frac{135}{143}$
18. 250,842
19. -305.96
20. $\frac{21}{32}$
21. 25
22. 8
23. 16
24. 49
25. 512
26. 100
27. 256
28. 243
29. 64
30. 81
31. 10,000
32. 125
33. 100,000
34. 343
35. 32
36. Let’s talk about it.

10^4= 10,000 10^8 = 100,000,000 10^3 = 1,000

Section 1.7

Scientific Notation

Boy, are those tough or what!? When dealing with 10, all you have to do is make sure you have that many zeros. This little fact allows scientists to write very large numbers in a very small way. For example, look at the number 70,000,000,000,000.

Instead of writing all of those zeros, we could (almost like expanded notation) write $7x10^{13}$.

Here is another:

$$4,000,000,000 = 4x10^9$$

Notice how all those decimal places just get sucked up into the exponent. It is **official scientific notation when the first number is between 1 and 10**.

Another:

$$53,400,000,000 = 5.34x10^{10}$$

Between 1-10

Here is a list that will probably help you to understand just a little bit better.

2,390,000 Decimal notation

$239,000x10$

$23,900x10^2$

$2,390x10^3$

$239x10^4$

$23.9x10^5$

$2.39x10^6$ Scientific notation

Do you see how each time we move the decimal place, we just keep track by increasing the exponent on the 10. They all represent the same number.

Here is a look at some numbers and their scientific notation:

300,000	$3x10^5$
30,000	$3x10^4$
3,000	$3x10^3$
300	$3x10^2$
30	$3x10^1$

What do you think comes next in the pattern?

3	$3x10^0$
.3	$3x10^{-1}$
.03	$3x10^{-2}$
.003	$3x10^{-3}$
.0003	$3x10^{-4}$

Since we have established that the exponent tells us how to move the decimal, notice how the **negative sign** means that we are moving the decimal the **other direction**, making it so that we can write really small numbers as well.

Also, almost as a side note, you are able to expand numbers (from Section 1.3) in a little bit different way. Here is how you could expand

20,300,000.709

Expanded: $2(10{,}000{,}000)+3(100{,}000)+7(10^{th})+9(1000^{th})$

Alternative: $2(10^7)+3(10^5)+7(10^{-1})+9(10^{-3})$

Please note how a **negative exponent** doesn't mean that we have a negative number. It just **means that we are behind the decimal for a very small fraction.**

Ok, arithmetic with scientific notation. With addition and subtraction, we still have to have like things.

Lining up the decimals means that you have to have the same exponent on the 10.

$$\begin{array}{r} 3.3x10^7 \\ +\ \underline{5.6x10^5} \end{array} \quad \xrightarrow{} \quad \text{Get like things} \quad \begin{array}{r} 330x10^5 \\ \underline{\ 5.6x10^5} \\ 335.6x10^5 \end{array}$$

Put it in scientific notation.

$$= 3.356x10^7$$

Here is how the summary of steps for Addition and Subtraction would go with Scientific Notation:

Addition/Subtraction of Scientific Notation

1. Set it up
   - 1.Get same exponent (left-up)
   - 2. Subtract with bigger on top.
2. Add/Subtract Columns
3. Carry or borrow by 10's.
4. Strongest number wins
5. Put into scientific notation (left-up).

Examples

Multiplication, also, isn't so bad:

When multiplying 10's just add the zeros.

$(3.2\text{x}10^{5})\text{x}(4.8\text{x}10^{12}) = 15.36\text{x}10^{17}$

But that isn't in scientific notation.

$= 1.536\text{x}10^{18}$ (scientific notation)

Ahh. Now it is.

One last one of multiplication

$$(3.42\text{x}10^{15})\text{x}(9\text{x}10^{5}) = 30.78\text{x}10^{20}$$
$$=3.078\text{x}10^{21}$$

Summary of Multiplication of Scientific Notation:

Multiplication of Scientific Notation

1. Multiply decimals.
2. Add exponents.
3. Put into scientific notation (left-up)

Examples

Section 1.7 Exercises

Subtract.

1.4

1. -13 – (-7) =

2. -15 - 12 =

3. 13 – 11 =

4. $\begin{array}{r} 8,105 \\ \underline{-4,982} \end{array}$

5. $\begin{array}{r} 2047.5 \\ \underline{-\ \ 93.72} \end{array}$

6. $10\frac{2}{13} - 7\frac{8}{13} =$

Multiply.

1.5

7. .271 x (-35) =

8. $\frac{13}{9} \times \frac{5}{9} =$

9. $\begin{array}{r} 4,125,000 \\ \underline{\text{x} \qquad\quad 8} \end{array}$

10. $\frac{3}{19} \times 6 =$

11. $\frac{15}{13} \times \frac{3}{10} =$

12. 351x475 =

Find.

1.6

13. $10^{13}=$

14. $6^{3}=$

15. $14^{2}=$

16. $13^{2}=$

17. $4^{3}=$

18. $9^{2}=$

Convert into scientific notation.

1.7

Example:
24,900,000 =

move the decimal
point 7 places

$$2\overbrace{4,900,000} = 2.49 \text{ x } 10^{7}$$

until the number is
between 1 and 10

19. 2,000,000

20. 4,300

21. 5,280,000,000

22. .000042

23. 85,000

24. .251

25. 730,000,000

26. .0000000321

27. .004

Convert into decimal notation.

Example:
$3.27x10^{-7}=$

move the decimal point -7 places

$3.27x10^{-7} = \overbrace{.000000}327$

28. $5.30x10^{9}$

29. $8.015x10^{7}$

30. $3.14x10^{5}$

31. $7x10^{-3}$

32. $6.9x10^{10}$

33. $9.54x10^{-5}$

34. $4.93x10^{-11}$

35. $3x10^{4}$

36. $2.71x10^{-17}$

Find.

Example:
$(8.2x10^{7})+(5.7x10^{6})$

Adjust, so you can line up the decimal point

$(8.2x10^{7}) + (5.7x10^{6}) = 82\ \ x10^{6}$

$\underline{+\ 5.7x\ 10^{6}}$

$87.7x10^{6}$

$8.77x10^{7}$

Must be the same to add.

Change back into standard scientific notation.

Example:
$(7.2x10^{6})x(3.1x10^{9})$

When multiplying, add the zeros.

$(7.2x10^{6})x(3.1x10^{9}) = 22.32x10^{15}$

Multiply the numbers

$2.232x10^{16}$

Change back into standard scientific notation.

37. $(5.30x10^{9})+(6.9x10^{10})$

38. $(8.015x10^{7})+(3.14x10^{5})$

39. $(5.30x10^{9}x(6.9x10^{10})$

40. $(9.54x10^{-5})+(7x10^{-3})$

41. $(8.01x10^{7})x(3.1x10^{5})$

42. $(9.54x10^{-5})x(7x10^{-3})$

43. $(7.24x10^{-7})+(7.1x10^{-8})$

44. $(4.05x10^{7})x(7.19x10^{-5})$

45. $(2.7x10^{-9})x(5x10^{-13})$

46. Preparation. What is the hardest kind of problem so far?

Answers:

1. -6

2. -27

3. 2

4. 3,123

5. 1,953.78

6. $2\frac{7}{13}$ or $\frac{7}{13}$

7. -9.485

8. $\frac{65}{81}$

9. 33,000,000

10. $\frac{18}{19}$

11. $\frac{9}{26}$

12. 166,725

13. 10,000,000,000,000

14. 216

15. 196

16. 169

17. 64

18. 81

19. $2 \text{x} 10^{6}$

20. $4.3 \text{x} 10^{3}$

21. $5.28 \text{x} 10^{9}$

22. $4.2 \text{x} 10^{-5}$

23. $8.5 \text{x} 10^{4}$

24. $2.51 \text{x} 10^{-1}$

25. $7.3 \text{x} 10^{8}$

26. $3.21 \text{x} 10^{-8}$

27. $4 \text{x} 10^{-3}$

28. 5,300,000,000

29. 80,150,000

30. 314,000

31. .007

32. 69,000,000,000

33. .0000954

34. .0000000000493

35. 30,000

36. .0000000000000000271

37. $7.43 \text{x} 10^{10}$

38. $9.38 \text{x} 10^{7}$

39. $3.657 \text{x} 10^{20}$

40. $7.0954 \text{x} 10^{-3}$

41. $2.4831 \text{x} 10^{13}$

42. $6.678 \text{x} 10^{-7}$

43. $7.95 \text{x} 10^{-7}$

44. $2.91195 \text{x} 10^{3}$

45. $1.35 \text{x} 10^{-21}$

46. Let’s talk about it.

Chapter Review and Summary:

Estimation
1. Round to the highest value.
2.Do the easy problem.

Addition/Subtraction of Decimals:
1. Set it up
2. Add/Subtract Columns
3. Carry or borrow by 10's
4. Stronger number wins. (+ or -)

Multiplication of Fractions
1. No common denominator. (Yippy!)
2. Multiply numerators.
3. Multiply denominators.
4. Simplify

Multiplication of Decimals
1. Multiply each place value
2. Carry by 10's
3. Add
4. Right size.

Multiplication of Scientific Notation
1. Multiply decimals.
2. Add exponents.
3. Put into scientific notation (left-up)

Addition/Subtraction of Scientific Notation
1. Set it up
2. Add/Subtract Columns
3. Carry or borrow by 10's.
4. Strongest number wins
5. Put into scientific notation (left-up).

Chapter 1 Review Exercises

Write the numbers in expanded form.

1.3

1. 54,023

2. .003507

3. 83.095

Add.

4. $\begin{array}{r} 2{,}578 \\ \underline{+\ 389.4} \end{array}$

5. $\begin{array}{r} 3{,}691 \\ \underline{+\quad 45} \end{array}$

6. $7\frac{5}{23}+14\frac{20}{23}=$

7. $3\frac{2}{9}+9\frac{8}{9}=$

8. $14\frac{6}{11}+5\frac{2}{11}=$

9. $\begin{array}{r} 27.471 \\ \underline{+\ 11.36} \end{array}$

1.4

Subtract.

10. -5 –18 =

11. -33 +15 =

12. -12 – (-37) =

13. $\begin{array}{r} 973 \\ \underline{-3{,}284} \end{array}$

14. $\begin{array}{r} 58.93 \\ \underline{-17.986} \end{array}$

15. $7\frac{4}{5}-18\frac{2}{5}=$

Find the final result.

16. Cost:$735
Discount:$41.79
Final Price:

17. Temp:-58.5° F
Change:18.7° colder
Final:

18. Altitude: -284 m
Rise: 558 m
Final:

1.5

Multiply.

19. 719x8 =

20. $\frac{5}{13}\times\frac{7}{3}=$

21. $\begin{array}{r} 4{,}127 \\ \underline{\text{x}\quad 9} \end{array}$

22. 34.8x.06 =

23. 392x74 =

24. $\frac{9}{5}\times\frac{7}{13}=$

Find the final result.

25. Cost: $5.77
Quantity: 17
Total:

26. Length: $\frac{7}{32}$ inch
Quantity: 3
Total length:

1.6

Find.

27. $10^7=$
28. $5^3=$
29. $15^2=$
30. $11^2=$
31. $2^3=$
32. $7^2=$

1.7

Convert into scientific notation.

33. 5,000,000
34. 4,900
35. 4,200,000,000
36. .00033
37. 19,000
38. .031

Convert into decimal notation.

39. $5.70x10^5$
40. $3.015x10^8$
41. $7.54x10^{13}$
42. $2x10^{-7}$
43. $5.9x10^{-4}$
44. $9.41x10^{-9}$

Find.

45. $(5.30x10^{-9})+(6.9x10^{-9})$
46. $(8.015x10^6)+(5.70x10^5)$
47. $(3.015x10^8)x(6.9x10^{10})$
48. $(9.54x10^{-5})+(5.9x10^{-4})$
49. $(8.01x10^7)x(3.1x10^5)$
50. $(9.54x10^{-5})x(7x10^{-3})$

Answers:

1. $5(10^4)+4(10^3)+2(10)+3(1)$
2. $3(10^{-3})+5(10^{-4})+7(10^{-6})$
3. $8(10)+3(1)+9(10^{-2})+5(10^{-3})$
4. 2,967.4
5. 3,736
6. $22\frac{2}{23}$
7. $13\frac{1}{9}$
8. $19\frac{8}{11}$
9. 38.831
10. -23
11. -18
12. 25
13. -2,311
14. 40.944
15. $-10\frac{3}{5}$
16. $693.21
17. -77.2° F
18. 274 m
19. 5,752
20. $\frac{35}{39}$
21. 37,143
22. 2.088
23. 29,008
24. $\frac{63}{65}$
25. $98.09
26. $\frac{21}{32}$
27. 10,000,000
28. 125
29. 225
30. 121
31. 8
32. 49
33. $5\text{x}10^6$
34. $4.9\text{x}10^3$
35. $4.2\text{x}10^9$
36. $3.3\text{x}10^{-4}$
37. $1.9\text{x}10^4$
38. $3.1\text{x}10^{-2}$
39. 570,000
40. 301,500,000
41. 75,400,000,000,000
42. .0000002
43. 59,000
44. .00000000941
45. $1.22\text{x}10^{-8}$
46. $8.585\text{x}10^6$
47. $2.08035\text{x}10^{19}$
48. $6.854\text{x}10^{-4}$
49. $2.4831\text{x}10^{13}$
50. $6.678\text{x}10^{-7}$

Chapter 1 Review Exercises (2)

Write the numbers in expanded form.

1.3

1. 54,023

2. .003507

3. 83.095

Add.

4. $\begin{array}{r} 2{,}578 \\ \underline{+\ 389.4} \end{array}$

5. $\begin{array}{r} 3{,}691 \\ \underline{+\quad 45} \end{array}$

6. $7\frac{5}{23}+14\frac{20}{23}=$

7. $3\frac{2}{9}+9\frac{8}{9}=$

8. $14\frac{6}{11}+5\frac{2}{11}=$

9. $\begin{array}{r} 27.471 \\ \underline{+\ 11.36} \end{array}$

1.4

Subtract.

10. -5 –18 =

11. -33 +15 =

12. -12 – (-37) =

13. $\begin{array}{r} 973 \\ \underline{-3{,}284} \end{array}$

14. $\begin{array}{r} 58.93 \\ \underline{-17.986} \end{array}$

15. $7\frac{4}{5}-18\frac{2}{5}=$

Find the final result.

16. Cost:$735
Discount:$41.79
Final Price:

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Final:

18. Altitude: -284 m
Rise: 558 m
Final:

1.5

Multiply.

19. 719x8 =

20. $\frac{5}{13}\times\frac{7}{3}=$

21. $\begin{array}{r} 4{,}127 \\ \underline{\text{x}\quad 9} \end{array}$

22. 34.8x.06 =

23. 392x74 =

24. $\frac{9}{5}\times\frac{7}{13}=$

Find the final result.

25. Cost: $5.77
Quantity: 17
Total:

26. Length: $\frac{7}{32}$ inch
Quantity: 3
Total length:

1.6

Find.

27. $10^7=$
28. $5^3=$
29. $15^2=$
30. $11^2=$
31. $2^3=$
32. $7^2=$

1.7

Convert into scientific notation.

33. 5,000,000
34. 4,900
35. 4,200,000,000
36. .00033
37. 19,000
38. .031

Convert into decimal notation.

39. $5.70\text{x}10^5$
40. $3.015\text{x}10^8$
41. $7.54\text{x}10^{13}$
42. $2\text{x}10^{-7}$
43. $5.9\text{x}10^{-4}$
44. $9.41\text{x}10^{-9}$

Find.

45. $(5.30\text{x}10^{-9})+(6.9\text{x}10^{-9})$
46. $(8.015\text{x}10^6)+(5.70\text{x}10^5)$
47. $(3.015\text{x}10^8)\text{x}(6.9\text{x}10^{10})$
48. $(9.54\text{x}10^{-5})+(5.9\text{x}10^{-4})$
49. $(8.01\text{x}10^7)\text{x}(3.1\text{x}10^5)$
50. $(9.54\text{x}10^{-5})\text{x}(7\text{x}10^{-3})$

Answers:

1. $5(10^4)+4(10^3)+2(10)+3(1)$

2. $3(10^{-3})+5(10^{-4})+7(10^{-6})$

3. $8(10)+3(1)+9(10^{-2})+5(10^{-3})$

4. 2,967.4

5. 3,736

6. $22\frac{2}{23}$

7. $13\frac{1}{9}$

8. $19\frac{8}{11}$

9. 38.831

10. -23

11. -18

12. 25

13. -2,311

14. 40.944

15. $-10\frac{3}{5}$

16. $693.21

17. -77.2° F

18. 274 m

19. 5,752

20. $\frac{35}{39}$

21. 37,143

22. 2.088

23. 29,008

24. $\frac{63}{65}$

25. $98.09

26. $\frac{21}{32}$

27. 10,000,000

28. 125

29. 225

30. 121

31. 8

32. 49

33. $5\text{x}10^6$

34. $4.9\text{x}10^3$

35. $4.2\text{x}10^9$

36. $3.3\text{x}10^{-4}$

37. $1.9\text{x}10^4$

38. $3.1\text{x}10^{-2}$

39. 570,000

40. 301,500,000

41. 75,400,000,000,000

42. .0000002

43. 59,000

44. .00000000941

45. $1.22\text{x}10^{-8}$

46. $8.585\text{x}10^6$

47. $2.08035\text{x}10^{19}$

48. $6.854\text{x}10^{-4}$

49. $2.4831\text{x}10^{13}$

50. $6.678\text{x}10^{-7}$

Chapter 1 Review Exercises (3)

1.5

Multiply.

1. 749x19 =

2. $\frac{7}{15}\times\frac{7}{4}=$

3. $\begin{array}{r}5{,}923\\ \underline{\text{x}\quad 7}\end{array}$

4. 415.92x.08 =

5. 295x5 =

6. $\frac{3}{5}\times\frac{2}{9}=$

Find the final result.

7. Cost: $45.69
Quantity: 7
Total:

8. Length: $\frac{7}{29}$ inch
Quantity: 4
Total length:

1.6

Find.

9. $10^{15}=$

10. $9^{3}=$

11. $14^{3}=$

12. $17^{2}=$

13. $2^{5}=$

14. $3^{5}=$

1.7

Convert into scientific notation.

15. 71,080,000

16. 6,500

17. 7,290,000,000

18. $273.5\text{x}10^{18}$

19. $.036\text{x}10^{9}$

20. $153.4\text{x}10^{-9}$

Convert into decimal notation.

21. $6.42\text{x}10^{-15}$

22. $7.1\text{x}10^{5}$

23. $8.54\text{x}10^{11}$

24. $8.209\text{x}10^{-4}$

25. $9.3\text{x}10^{-1}$

26. $3.41\text{x}10^{-9}$

Find.

27. $(7.31\text{x}10^{-9})+(6.9\text{x}10^{-9})$

28. $(8.24\text{x}10^{7}) - (5.7\text{x}10^{6})$

29. $(5\text{x}10^{8})\text{x}(2.1\text{x}10^{19})$

30. $(3.54\text{x}10^{6})+(9.74\text{x}10^{7})$

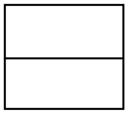
31. $(3.2\text{x}10^{4})\text{x}(9.41\text{x}10^{-11})$

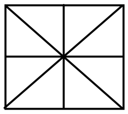
32. $(7.54\text{x}10^{-5}) - (2\text{x}10^{-5})$

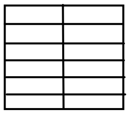
Preparation.

33. I have a model that is 7 inches wide and 10 inches tall. How big would it be if I made it 5 times as big?

34. Copy each square onto your paper. Color in half of each square.







Answers:

1. 14,231

2. $\frac{49}{60}$

3. 41,461

4. 33.2736

5. 1,475

6. $\frac{6}{45}$ or $\frac{2}{15}$

7. $319.83

8. $\frac{36}{37}$ inch

9. 1,000,000,000,000,000

10. 729

11. 2,744

12. 289

13. 32

14. 243

15. $7.108 \mathrm{x} 10^{7}$

16. $6.5 \mathrm{x} 10^{3}$

17. $7.29 \mathrm{x} 10^{9}$

18. $2.735 \mathrm{x} 10^{20}$

19. $3.6 \mathrm{x} 10^{7}$

20. $1.534 \mathrm{x} 10^{-7}$

21. .00000000000000642

22. 710,000

23. 854,000,000,000

24. .0008209

25. .93

26. .00000000341

27. $1.421 \mathrm{x} 10^{-8}$

28. $7.67 \mathrm{x} 10^{7}$

29. $1.05 \mathrm{x} 10^{28}$

30. $1.0094 \mathrm{x} 10^{8}$

31. $3.0112 \mathrm{x} 10^{-6}$

32. $5.54 \mathrm{x} 10^{-5}$

33. 35 inches by 50 inches

34. Let’s talk about it.

Chapter 2:

MORE ARITHMETIC

Overview

Section 2.1

Equivalent Fractions

Now that you can multiply most things together, we will work on taking them back apart. For example, we know that

$$5x6 = 30$$

With that in mind, we should be able to do:

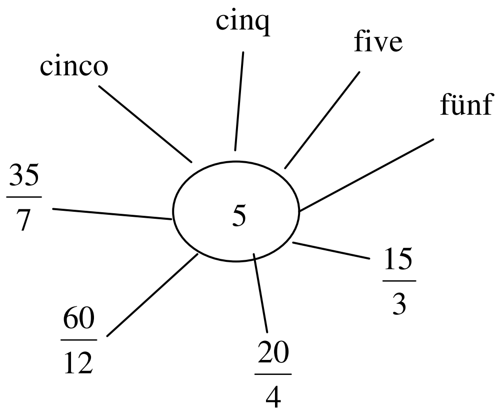
$6\overline{)30}$ or $\frac{30}{6}$ or $30 \div 6$ which all equal 5

A couple more:

$$\frac{30}{15} = 2 \qquad \frac{20}{5} = 4 \qquad \frac{36}{4} = 9$$

In connection with the multiplication tables, you should commit to memorization all of the division problems that go along with them. Trust me, you are going to use them. . . a lot.

With division as a tool in our toolbox, we are finally ready to discuss names. Numbers have this funny way of changing masks. Let me explain. If we take the number 5, in French is cinq; Spanish, cinco; and in German, fünf.



But there are also other names for 5. Sticking to English, we have 15/3, 20/4, and 60/12. All of them are simply different masks for the number 5.

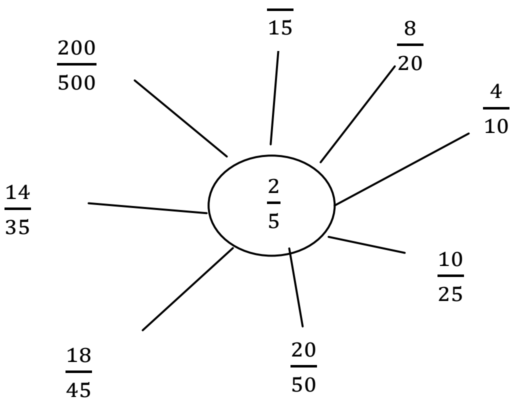
One number, in particular needs to be mentioned along with some of its other names.

$$1=\frac{3}{3} \qquad 1=\frac{4}{4} \qquad 1=\frac{10}{10} \qquad 1=\frac{23}{23}$$

Do you remember the easiest times tables problems? Like 7x1=7 or 15x1= 15. Multiplication by 1 never changed the number, so if we take any number like $\frac{2}{5}$ and multiply it by 1(with a different name) we get:

$$\frac{2}{5} x \frac{3}{3} = \frac{6}{15} \quad \text{or} \quad \frac{2}{5} x \frac{4}{4} = \frac{8}{20}$$

As a picture like above, we have the following:



Since we multiplied by 1 each time, we actually didn't change the number at all, but simply changed its name. Notice it can also go the other way:

$\frac{14}{20}$ is the same as $\frac{14}{20} = \frac{7}{10} x \frac{2}{2}$

$$\frac{14}{20} = \frac{7}{10}$$

$\frac{20}{25}$ is the same as $\frac{20}{25} = \frac{4}{5} x \frac{5}{5}$

$$\frac{20}{25} = \frac{4}{5}$$

This process of **reducing fraction down until there isn't anything common is called simplifying.**

A couple of illustrations:

$$\frac{28}{35} = \frac{4}{5} \qquad \frac{16}{32} = \frac{1}{2}$$

What about simplifying something with really big numbers like:

$$\frac{144}{192}$$

There are basically three ways to do this simplification:

1) Observation: observe what number is common and take it out.
2) Find a small number that goes into both of them, take it out, then repeat.
3) Break the numbers up into their **prime factors** to see exactly what will come out.

Method 1 – Observation is usually very handy for small numbers where the multiplication tables have been memorized – not so keen for this example.

Method 2 – Small chunks can readily be taken out:

$$\frac{144}{192}=\frac{72}{96}=\frac{36}{48}=\frac{6}{8}=\frac{3}{4}$$

Method 3 – Prime factorization – We are going to break up numbers to see what they are made of. It is kind of like the genetic code of the numbers

$$\frac{144}{192}=\frac{2x2x2x2x3x3}{2x2x2x2x2x2x3}\qquad=\qquad\frac{1x1x1x1x3x1}{2x2}\qquad\frac{3}{4}$$

Notice now that each time there is a $\frac{2}{2}$ or a $\frac{3}{3}$, it counts as a 1. One times anything doesn't change it so all of the 1's don't count.

Here is another example to simplify:

$$\frac{168}{180}=\frac{2x2x2x3x7}{2x2x3x3x5}\qquad=\qquad\frac{2x7}{3x5}=\frac{14}{15}$$

Prime factorization is really useful when you need to see what a number is made up of. We will be using it again.

One may ask why all of this practice and talk about taking a fraction, multiplying it by 1 a bunch of times until it looks different, and then simplifying it back down. A well timed question. For the sake of reference if we create an equivalent fraction by dividing by the same number on top and bottom, we will call it **simplifying.** If we create an equivalent fraction by multiplying on top and bottom, we will call it **complexifying.** (If there is another word out there that is real and means the same thing, I haven't found it yet.)

Another example:

$\frac{2}{3},\frac{4}{6},\frac{6}{9},\frac{8}{12},\frac{10}{15},\frac{20}{30},\frac{40}{60},\frac{2000}{3000}$ are all names for $\frac{2}{3}$. We have just **multiplied by a different "1"** to keep the same number, but change how it looked. Likewise all of these different fractions are equivalent to each other and **simplify** down to $\frac{2}{3}$.

Examples

Section 2.1 Exercises

1.7

Convert into scientific notation.

1. 61,070 **2.** 56,500 **3.** 9,390,000,000

4. $27.71x10^{14}$ **5.** $.0034x10^{7}$ **6.** $154.4x10^{-7}$

Convert into decimal notation.

7. $3.25x10^{-15}$ **8.** $7.8x10^{9}$ **9.** $3.74x10^{-8}$

Find.

10. $(2.5x10^{-10})+(4.9x10^{-9})$ **11.** $(7.3x10^{7})$ - $(4.1x10^{7})$ **12.** $(4.3x10^{18})x(2.1x10^{79})$

13. $(4.283x10^{6})+(9x10^{4})$ **14.** $(4.7x10^{4})x(9.71x10^{-11})$ **15.** $(7.54x10^{-4})$ - $(2x10^{-5})$

2.1

Find 4 different names for each fraction:

Example:

$$\frac{3}{11} \dashrightarrow \frac{3}{11},\frac{6}{22},\frac{9}{33},\frac{12}{44},\frac{15}{55},\frac{30}{110},\frac{3,000}{11,000}\ldots$$

16. $\frac{2}{3}$ **17.** $\frac{4}{5}$ **18.** $\frac{5}{8}$ **19.** $\frac{7}{13}$

20. $\frac{9}{10}$ **21.** $\frac{2}{7}$ **22.** 7 **23.** $\frac{5}{6}$

24. 3 **25.** $\frac{4}{9}$ **26.** $\frac{3}{5}$ **27.** $\frac{8}{15}$

Simplify each fraction.

Example:

$$\frac{36}{96} \dashrightarrow \frac{36}{96}=\frac{\cancel{2}x\cancel{2}x3x\cancel{3}}{\cancel{2}x\cancel{2}x2x2x2x\cancel{3}}=\frac{3}{2x2x2}=\frac{3}{8}$$

Assignment 2.1

28. $\frac{30}{75}$

29. $\frac{7}{35}$

30. $\frac{16}{40}$

31. $\frac{28}{84}$

32. $\frac{36}{54}$

33. $\frac{45}{360}$

Create each fraction with a denominator of 72.

Example:

$$\frac{2}{3} \quad \dashrightarrow \quad \frac{2}{3}\cdot\frac{24}{24}=\frac{48}{72}$$

34. $\frac{5}{6}$

35. $\frac{3}{8}$

36. $\frac{1}{24}$

37. $\frac{7}{12}$

38. $\frac{5}{9}$

39. $\frac{31}{36}$

Create each fraction with a denominator of 54.

Example:

$$\frac{2}{3} \quad \dashrightarrow \quad \frac{2}{3}\cdot\frac{18}{18}=\frac{36}{54}$$

40. $\frac{5}{6}$

41. $\frac{1}{2}$

42. $\frac{23}{27}$

43. $\frac{7}{18}$

44. $\frac{5}{9}$

45. $\frac{1}{6}$

Preparation.

46. $\frac{1}{3}+\frac{1}{2}=$

Find 4 different names for $\frac{1}{3}$ and 4 different names for $\frac{1}{2}$. Find the names that have the same denominators. Add them. What do you get?

Answers:

1. $6.107\text{x}10^{4}$

2. $5.65\text{x}10^{4}$

3. $9.39\text{x}10^{9}$

4. $2.771\text{x}10^{15}$

5. $3.4\text{x}10^{4}$

6. $1.544\text{x}10^{-5}$

7. .00000000000000325

8. 7,800,000,000

9. .0000000374

10. $5.15\text{x}10^{-9}$

11. $3.2\text{x}10^{7}$

12. $9.03\text{x}10^{97}$

13. $4.373\text{x}10^{6}$

14. $4.5637\text{x}10^{-6}$

15. $7.34\text{x}10^{-4}$

16. $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{20}{30}, others...$

17. $\frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \frac{28}{35}, \frac{400}{500}, others...$

18. $\frac{10}{16}, \frac{15}{24}, \frac{25}{40}, \frac{35}{56}, \frac{45}{72}, \frac{55}{88}, others...$

19. $\frac{14}{26}, \frac{21}{39}, \frac{28}{52}, \frac{35}{65}, \frac{42}{78}, others...$

20. $\frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \frac{45}{50}, \frac{90}{100}, others...$

21. $\frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \frac{10}{35}, \frac{2000}{7000}, others...$

22. $\frac{14}{2}, \frac{21}{3}, \frac{28}{4}, \frac{35}{5}, \frac{63}{9}, others...$

23. $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{75}{90}, others...$

24. $\frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5}, \frac{27}{9}, others...$

25. $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}, \frac{40}{90}, \frac{44}{99}, others...$

26. $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{24}{40}, \frac{39}{65}, others...$

27. $\frac{16}{30}, \frac{24}{45}, \frac{32}{60}, \frac{48}{90}, others...$

28. $\frac{2}{5}$

29. $\frac{1}{5}$

30. $\frac{2}{5}$

31. $\frac{1}{3}$

32. $\frac{2}{3}$

33. $\frac{1}{8}$

34. $\frac{60}{72}$

35. $\frac{27}{72}$

36. $\frac{3}{72}$

37. $\frac{42}{72}$

38. $\frac{40}{72}$

39. $\frac{65}{72}$

40. $\frac{45}{54}$

41. $\frac{27}{54}$

42. $\frac{46}{54}$

43. $\frac{21}{54}$

44. $\frac{30}{54}$

45. $\frac{9}{54}$

46. In class . . .

Section 2.2

Unlike fractions

Now, to answer the question of why we would simplify. We **simplify** so that we can communicate effectively with others. For every number, there are at least a billion other names for it. Can you imagine how confused you might be if when you asked a friend their age, they told you that they were $\frac{255}{15}$ years old instead of 17? Or how about if the recipe in a cookbook told you to add $\frac{141}{188}$ cups of sugar instead of just saying $\frac{3}{4}$. Simplified is just standard communication. Not to mention, if you went around using those other fractions, people might think you were weird. That is why we simplify.

Ahhh, but why do we complexify? To see the answer, we look at some of the old addition problems we did:

$$\frac{1}{5}+\frac{3}{5}=\frac{4}{5} \quad \text{piece of cake}$$

$$\frac{2}{23}+\frac{7}{23}=\frac{9}{23} \quad \text{no problem}$$

We have learned that we can only add like things! What if the two things that we were trying to add **weren't** alike? For example:

3apples + 4guava =

Hah! We can't do it. Of course there are some of you out there that are clever and would say – sure we can, there are 7 pieces of fruit. Well, you have just cracked the secret! We still can only **add like things**, so in order to do that, we refer to what we have by what they both have in common:

3apples + 4 guava =
3 fruit + 4 fruit = 7 fruit

Now with numbers.

$$\frac{1}{2}+\frac{1}{3}= \quad \text{Argh, they aren't alike.}$$

Remember that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}\ldots$ are all the same thing.

And that $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}$ are all the same as well.

We could rewrite our problem to be:

$$\frac{1}{2}+\frac{1}{3}=$$

$$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

piece of cake.

That is the power of complexifying! Since each number has a billion names, you need to pick the names that will allow you to add like things.

Here is another one:

$$\begin{array}{ccc} \frac{1}{6} & + & \frac{5}{8} \quad = \\ \frac{2}{12} & & \frac{10}{16} \\ \frac{3}{18} & & \boxed{\frac{15}{24}} \\ \boxed{\frac{4}{24}} & + & \frac{20}{32} \\ \frac{5}{30} & & \frac{25}{40} \end{array}$$

$$\frac{4}{24}+\frac{15}{24}=\frac{19}{24}$$

This can get kind of tedious, so we start to look for some way of making the denominators, the bottoms of the fractions, the same. Here are the major ways of doing it:

1) Observation – If it easy to see what number both denominators can be made into, then do it.

2) Multiplying the two numbers will always get you a common denominator, but not always the smallest one.

3) Prime factorization – especially useful with larger numbers – I told you we would come back to this.

Examples to illustrate each one of these:

1)Observation – kind of easy to see common denominators

$$\frac{2}{3}+\frac{1}{6}= \qquad \frac{1}{6}+\frac{5}{9}= \qquad \frac{1}{3}+\frac{1}{6}=$$

$$\frac{4}{6}+\frac{1}{6}=\frac{5}{6} \qquad \frac{3}{18}+\frac{10}{18}=\frac{13}{18} \qquad \frac{2}{6}+\frac{1}{6}=\frac{3}{6}$$

Don't forget to simplify

$$=\frac{1}{2}$$

Examples

2) Multiplying

5x6 = 30

Example:

$$\frac{3}{5}+\frac{1}{6}=$$

$$\frac{18}{30}+\frac{5}{30}=\frac{23}{30}$$

Example:

$$\frac{5}{8}+\frac{1}{12}=$$

$$\frac{60}{96}+\frac{8}{96}=\frac{68}{96}$$

$$=\frac{17}{24}$$

Sometimes there is a lot of simplifying to do.

3)Prime factorization – this takes a little more practice, but is so much more powerful. Let us first get a hold on what this "genetic code" really tells us.

Here is a list of the multiples of 10 and the prime factorizations of them.

10	2x5
2x10 = 20	2x2x5
3x10 = 30	2x3x5
4x10 = 40	2x2x2x5
5x10 = 50	2x5x5
6x10 = 60	2x2x3x5
7x10 = 70	2x5x7
8x 10 = 80	2x2x2x2x5
9x10 = 90	3x3x2x5
10 x10 =100	2x2x5x5

Now you should look at these numbers like you would look at a specimen under the microscope. Every single one of them contains a 2 and a 5 which is the code for 10. By analyzing them you can see the exact numbers that multiply to get them. For example:

2x3x3x5x7x7x11x19

Even though we don't know what the number actually is, I can tell you that it is a multiple of 10 (it has a 2 and a 5). It is also a multiple of 21(there is a 3 and a 7).

It is also a multiple of 6 (has a 2 and 3) and a multiple of 9 (has a 3x3 in it). Are you starting to feel the power of cutting a number open and laying it down to the bare bones?

Now, to construct the smallest number that both denominators can go into: We call this the Least Common Multiple or Least Common Denominator. Let's find the Least Common Multiple of

27 and 36.

27 = 3x3x3 36 = 2x2x3x3

which means that any number with 3x3x3 in it will be a multiple of 27. Likewise, any number that has 2x2x3x3 in it will be a multiple of 36.

Obviously 3x3x3x2x2x3x3 is a multiple of both of them, but is it the ***least*** one?

The smallest would need to be:

3x3x3x2x2

= 108

It has both numbers completely hidden inside with no overlap or extra. That is our baby.

Try another one. Find the Least Common Multiple of:

150 and 20

2x3x5x5 2x2x5

Well, we know it has to have at least 2x3x5x5 so that 150 will go into it. Will 20 go into it? Not yet. It looks like we need one more 2.

End result: 2x2x3x5x5

That should help you do huge problems like:

$\frac{15}{28}+\frac{31}{64}=$ we find the LCM of 28 and 64

2x2x7 2x2x2x2x2x2

LCM = 2x2x2x2x2x2x7

= 448

Change each fraction by multiplying by 1

$$\frac{15}{28}\times\frac{16}{16}+\frac{13}{64}\times\frac{7}{7}$$

$$\frac{240}{448}+\frac{217}{448}=\frac{457}{448}$$

$$=1\frac{9}{448}$$

Examples

Addition/Subtraction with Fractions

1. Set it up. (Subtract with bigger on top)
2. Get common denominators.
    1. Observation
    2. Multiply the denominators
    3. Prime factorization
3. Add/Subtract numerators.
4. Carry or borrow by denominator.
5. Strongest number wins.
6. Simplify.
    1. Observation
    2. Prime factorization

Use with Fraction Multiplication

If you remember multiplication of fractions, it is really difficult to multiply numbers like $5\frac{2}{7}$. We can't really just multiply tops and bottoms here. We need to turn it into something else. Well, you now have the complete capability to do that:

As you practice you can get a quick way of converting these. Notice 5x7+2 = 37.

$$5\tfrac{2}{7} = 5 + \tfrac{2}{7}$$

Common denominator is 7

$$= \tfrac{35}{7} + \tfrac{2}{7} = \tfrac{37}{7}$$

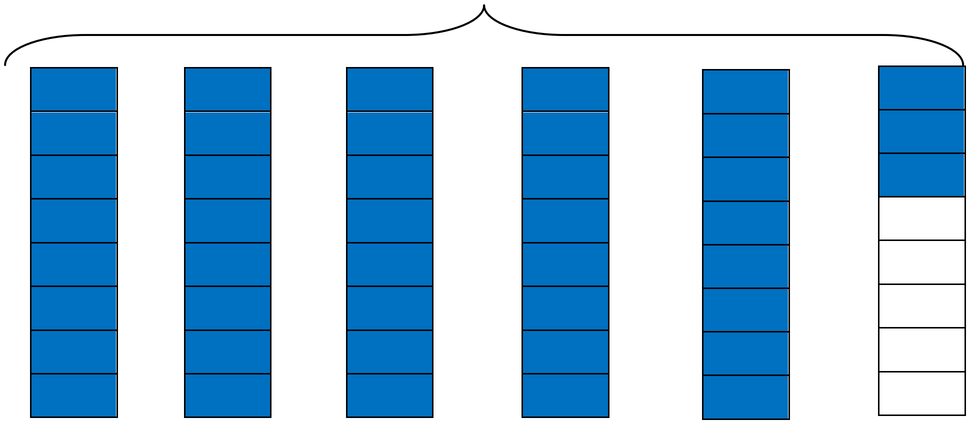
Sometimes, like with addition, it will help to leave it as $5\frac{2}{7}$. On the other hand, when we multiply fractions, we multiply across the top and across the bottom. However, with $5\frac{2}{7}$, we can see that the 5 isn't on the top or the bottom, so to be able to multiply, you will need to write it as $\frac{37}{7}$.

Either way you should be able to move from one to the other without much difficulty. Just to make sure we are all on the same page, we give these two different ways of writing a fraction distinct names:

Improper fractions have numerators that are larger than the denominator, while **mixed numbers** don't.

Mixed Number		Improper Fraction
$5\frac{3}{8}$		$\frac{43}{8}$
$7\frac{1}{5}$	=	$\frac{36}{5}$
$3\frac{7}{8}$		$\frac{31}{8}$
$1\frac{18}{19}$		$\frac{37}{19}$

Illustration of $5\frac{3}{8} \qquad = \frac{43}{8}$



You should notice that to go backward and change an improper fraction back to a mixed number, we just do the division. The only thing to watch is what happens with the remainder.

Here are a couple of examples:

$$\frac{43}{8} =$$

$$\begin{array}{r} 5 \\ 8\overline{)43} \\ -\underline{40} \\ 3 \end{array}$$

which means 5 R 3 or in other words $5\frac{3}{8}$

Upon quite a bit of practice, you should be able to now add, subtract or multiply any fraction at all.

Section 2.2 Exercises

1.7

Convert into scientific notation.

1. 4,659 **2.** .00067 **3.** 9,240,000

Find.

4. $(2.3\text{x}10^{-10})+(7.9\text{x}10^{-11})$ **5.** $(9.5\text{x}10^{7}) - (8.1\text{x}10^{7})$ **6.** $(2.5\text{x}10^{12})\text{x}(2.5\text{x}10^{6})$

7. $(4.351\text{x}10^{6})+(9\text{x}10^{4})$ **8.** $(4.3\text{x}10^{12})\text{x}(9.2\text{x}10^{-5})$ **9.** $(3.57\text{x}10^{-4}) - (1\text{x}10^{-5})$

2.1

Find 4 different names for each fraction:

10. $\frac{4}{7}$ **11.** $\frac{2}{9}$ **12.** $\frac{7}{10}$ **13.** $\frac{7}{8}$

Simplify each fraction.

14. $\frac{32}{50}$ **15.** $\frac{27}{30}$ **16.** $\frac{16}{28}$

17. $\frac{90}{120}$ **18.** $\frac{13}{42}$ **19.** $\frac{12}{360}$

Create each fraction with a denominator of 48.

20. $\frac{5}{6}$ **21.** $\frac{3}{8}$ **22.** $\frac{1}{24}$

Create each fraction with a denominator of 64.

23. $\frac{5}{8}$ **24.** $\frac{3}{16}$ **25.** $\frac{1}{2}$

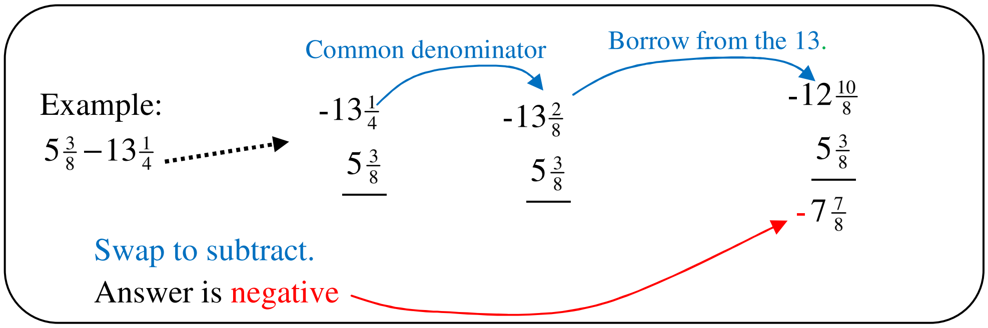
2.2

Add or Subtract. Simplify.

Example:

$\frac{1}{2}+\frac{3}{7}=$

$\frac{7}{14}+\frac{6}{14}=\frac{13}{14}$



26. $\frac{2}{5}+\frac{1}{3}=$

27. $\frac{1}{4}+\frac{3}{8}=$

28. $\frac{5}{6}-\frac{3}{8}=$

29. $\frac{3}{8}+\frac{7}{12}=$

30. $2\frac{3}{4}+7\frac{1}{6}=$

31. $5\frac{5}{7}-3\frac{2}{3}=$

32. $7\frac{4}{5}-\frac{9}{10}=$

33. $\frac{7}{8}+9\frac{2}{3}=$

34. $12\frac{11}{16}-19\frac{3}{8}=$

35. $\frac{1}{32}-\frac{11}{12}=$

36. $5\frac{2}{3}+3\frac{1}{5}$

37. $\frac{5}{16}+\frac{5}{24}=$

Fill out the table.

	Mixed	Improper
38.	$5\frac{2}{3}$	
39.	$7\frac{1}{5}$	
40.		$\frac{43}{9}$
41.		$\frac{51}{7}$
42.	$-8\frac{1}{4}$	
43.		$-\frac{16}{3}$
44.		$\frac{7}{6}$

Preparation.

45.
$12\div 2=$
$12\times\frac{1}{2}=$

46.
$20\div 4=$
$20\times\frac{1}{4}=$

47.
$14\div\frac{2}{3}=$
$14\times\frac{3}{2}=$

Answers:

1. $4.659\text{x}10^{3}$

2. $6.7\text{x}10^{-4}$

3. $9.24\text{x}10^{6}$

4. $3.09\text{x}10^{-10}$

5. $1.4\text{x}10^{7}$

6. $6.25\text{x}10^{18}$

7. $4.441\text{x}10^{6}$

8. $3.956\text{x}10^{8}$

9. $3.47\text{x}10^{-4}$

10. $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{40}{70}$, *others...*

11. $\frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \frac{200}{900}$, *others...*

12. $\frac{14}{20}, \frac{21}{30}, \frac{35}{50}, \frac{70}{100}$, *others...*

13. $\frac{14}{16}, \frac{21}{24}, \frac{49}{56}, \frac{84}{96}$, *others...*

14. $\frac{16}{25}$

15. $\frac{9}{10}$

16. $\frac{4}{7}$

17. $\frac{3}{4}$

18. $\frac{13}{42}$

19. $\frac{1}{30}$

20. $\frac{40}{48}$

21. $\frac{18}{48}$

22. $\frac{2}{48}$

23. $\frac{40}{64}$

24. $\frac{12}{64}$

25. $\frac{32}{64}$

26. $\frac{11}{15}$

27. $\frac{5}{8}$

28. $\frac{11}{24}$

29. $\frac{23}{24}$

30. $9\frac{11}{12}$

31. $2\frac{1}{21}$

32. $6\frac{9}{10}$

33. $10\frac{13}{24}$

34. $-6\frac{11}{16}$

35. $-\frac{85}{96}$

36. $8\frac{13}{15}$

37. $\frac{25}{48}$

38. $\frac{17}{3}$

39. $\frac{36}{5}$

40. $4\frac{7}{9}$

41. $7\frac{2}{7}$

42. $-\frac{37}{4}$

43. $-5\frac{1}{3}$

44. $1\frac{1}{6}$

45. In class.

46. In class.

47. In class.

Section 2.3

Fraction division

The only thing left to master is that of division. Let’s do fractions first. Try the following problems:

$$12 \div 2 = 6$$

$$12\text{x}\tfrac{1}{2} = 6$$

Did you notice that they both start with 12 and end with 6? Hmmm . . .

Here is another one:

$$36 \div 9 = 4$$

$$36\text{x}\tfrac{1}{9} = 4$$

Same thing. It is interesting to note that division and multiplication can end up at the same result. In fact, they are exact opposites of each other. 9 and $\frac{1}{9}$ are kind of opposite of each other as well. We call this relationship **multiplicative inverses or reciprocals**. Here are some examples:

Number	reciprocal
7	$\frac{1}{7}$
$\frac{4}{9}$	$\frac{9}{4}$
$-\frac{1}{3}$	-3

The cool thing about division is that you can change it into multiplication by using the reciprocal. For example:

$9 \div \frac{3}{4} =$ is kind of tough until you change it to multiplication.

$$9 \div \tfrac{3}{4} = 9x\tfrac{4}{3}$$

$$= \tfrac{9x4}{1x3} = \tfrac{36}{3} = 12$$

Division of Fractions

1. Multiply by reciprocal

Section 2.3 Exercises

2.1

Find 4 different names for each fraction:

1. $\frac{3}{7}$ **2.** $\frac{2}{3}$ **3.** $\frac{7}{11}$ **4.** $\frac{4}{9}$

Simplify each fraction.

5. $\frac{36}{52}$ **6.** $\frac{27}{36}$ **7.** $\frac{16}{56}$

8. $\frac{10}{12}$ **9.** $\frac{15}{45}$ **10.** $\frac{120}{280}$

Create each fraction with a denominator of 36.

11. $\frac{1}{6}$ **12.** $\frac{5}{9}$ **13.** $\frac{10}{12}$

2.2

Add or Subtract. Simplify.

14. $\frac{2}{5}+\frac{2}{3}=$ **15.** $\frac{1}{4}+\frac{5}{8}=$ **16.** $\frac{7}{30}-\frac{3}{25}=$

17. $\frac{1}{3}+\frac{7}{12}=$ **18.** $13\frac{3}{4}+4\frac{5}{6}=$ **19.** $9\frac{7}{10}-3\frac{1}{5}=$

20. $3\frac{9}{14}-6\frac{6}{7}=$ **21.** $4\frac{2}{7}+9\frac{2}{3}=$ **22.** $12\frac{5}{8}-9\frac{3}{4}=$

Fill out the table.

	Mixed	Improper
23.	$-7\frac{8}{9}$	
24.	$3\frac{1}{5}$	
25.		$\frac{43}{8}$
26.		$\frac{51}{4}$

Find the multiplicative inverse or reciprocal of each number.

Example:

$\frac{5}{8} \rightarrow \frac{8}{5}$

2.3

27. $\frac{4}{7}$ **28.** $\frac{2}{9}$ **29.** $-\frac{7}{10}$ **30.** $\frac{7}{8}$

31. $-\frac{5}{6}$

32. 13

33. $\frac{13}{42}$

34. $\frac{7}{3}$

Divide.

Example:

$2\frac{3}{8} \div \frac{4}{5} =$	
$2\frac{3}{8} \times \frac{5}{4} =$	Multiply by reciprocal
$\frac{19}{8} \times \frac{5}{4} =$	Change to improper fraction
$\frac{19}{8} \times \frac{5}{4} = \frac{95}{32}$ or $2\frac{31}{32}$	Multiply straight across.

35. $\frac{2}{5} \div \frac{1}{3} =$

36. $\frac{1}{4} \div \frac{3}{8} =$

37. $\frac{5}{6} \div \frac{3}{8} =$

38. $\frac{3}{8} \div \frac{7}{12} =$

39. $2\frac{3}{4} \div 7\frac{1}{6} =$

40. $5\frac{5}{7} \div 3\frac{2}{3} =$

41. $7\frac{4}{5} \div \frac{9}{10} =$

42. $\frac{7}{8} \div 9\frac{2}{3} =$

43. $2\frac{1}{6} \div \frac{3}{8} =$

Preparation.

44. Simplify the following fraction: $\frac{15000}{17000}$

45. Re-write this fraction with a denominator of 13.

$$\frac{7}{1.3}$$

Answers:

1. $\frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \frac{21}{49}$, *others...*

2. $\frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{12}{18}$, *others...*

3. $\frac{14}{22}, \frac{21}{33}, \frac{28}{44}, \frac{35}{55}$, *others...*

4. $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}, \frac{28}{63}$, *others...*

5. $\frac{9}{13}$

6. $\frac{3}{4}$

7. $\frac{2}{7}$

8. $\frac{5}{6}$

9. $\frac{1}{3}$

10. $\frac{3}{7}$

11. $\frac{6}{36}$

12. $\frac{20}{36}$

13. $\frac{30}{36}$

14. $1\frac{1}{15}$ or $\frac{16}{15}$

15. $\frac{7}{8}$

16. $\frac{17}{150}$

17. $\frac{11}{12}$

18. $18\frac{7}{12}$

19. $6\frac{1}{2}$

20. $-3\frac{3}{14}$

21. $13\frac{20}{21}$

22. $2\frac{7}{8}$

23. $-\frac{71}{9}$

24. $\frac{16}{5}$

25. $5\frac{3}{8}$

26. $15\frac{1}{2}$

27. $\frac{7}{4}$ or $1\frac{3}{4}$

28. $\frac{9}{2}$ or $4\frac{1}{2}$

29. $-\frac{10}{7}$ or $-1\frac{3}{7}$

30. $\frac{8}{7}$

31. $-\frac{6}{5}$ or $-1\frac{1}{5}$

32. $\frac{1}{13}$

33. $\frac{42}{13}$ or $3\frac{3}{13}$

34. $\frac{3}{7}$

35. $\frac{6}{5}$ or $1\frac{1}{5}$

36. $\frac{2}{3}$

37. $\frac{20}{9}$ or $2\frac{2}{9}$

38. $\frac{9}{14}$

39. $\frac{33}{86}$

40. $\frac{120}{77}$ or $1\frac{43}{77}$

41. $\frac{26}{3}$ or $8\frac{2}{3}$

42. $\frac{21}{232}$

43. $\frac{52}{9}$ or $5\frac{7}{9}$

44. In class.

45. In class.

Section 2.4

Division

Division of Decimals and integers isn't too bad for itself, but it just might get a little tedious. Before you do this you should have had some practice at doing long division.

Here let us take some time to solidify long division. The theory behind long division is that you just take it in individual chunks that are easy to manage. Do you remember how with multiplication, we just separated the number, did the multiplication and then added everything together? Well, we are going to undo that process. Let's look closer with just a few examples:

5 $8\overline{)429}$	We know that 8 goes into 42 about 5 times.
5 $8\overline{)429}$ $\underline{-40}$	Multiply 5x8 and subtract.
53 $8\overline{)429}$ $\underline{-40}\downarrow$ 29	Bring down the 9 to continue on. 8 goes into 29 about 3 times.
53 $8\overline{)429}$ $\underline{-40}$ 29 $\underline{-24}$ 5	Multiply 3x8 and subtract.
	8 doesn't go into 5 (remainder)

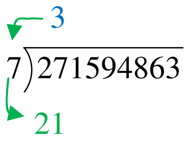
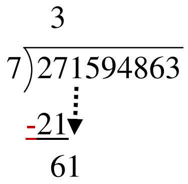
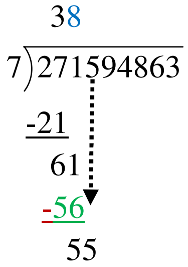
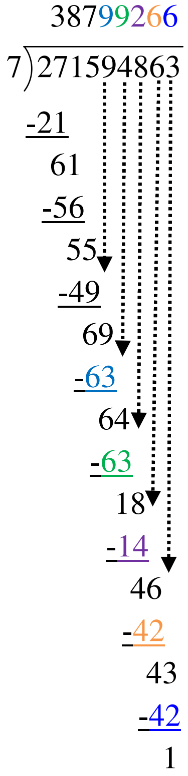
Which means that $429 \div 8 = 53$ R 5

or in other words $429 \div 8 = 53\frac{5}{8}$

Can you see that when we multiplied things together, we multiplied one column at a time and carried the extra to the next column and slowly worked our way from the right to the left. Division goes exactly opposite.

Now we slowly work from the left to the right knocking off each column and subtracting to get the next column.

Let’s do a big example:

	7 goes into 27 about 3 times Multiply 3x7 = 21
	Subtract. Bring down the next column
	7 goes into 61 about 8 times Multiply 8x7 = 56 Subtract. Bring down next column
	Continue to repeat the process until all of the columns are used up.

$271{,}594{,}863 \div 7 = 38{,}799{,}266$ R 1 or $38{,}799{,}266\frac{1}{7}$

Sometimes you may encounter something like this:

$$5875 \div 22$$

So we should definitely work one of these (where you are dividing by 2 or three digits) out as we cover this topic:

2 22)5875 44	22 goes into 58 about 2 times. Multiply 2x22 = 44
2 22)5875 -44 147	Subtract. Bring down the next column
27 22)5875 -44 147 ~~154~~	22 goes into 147 about ???? times. Let's estimate. 2 goes into 14 about 7 times – try that. Multiply 22x7 = 154 Oops, a little too big
26 22)5875 -44 147 -132 155	Since 7 was a little too big, try 6. Multiply 6x22 = 132 Subtract. Bring down the next column.
267 22)5875 -44 147 -132 155 -154 1	22 goes into 155 about ????? times. Estimate. 2 goes into 15 about 7 times. Try 7 Multiply 22x7 = 154. It worked. Subtract. Remainder 1

$$5875 \div 22 = 267 \text{ R } 1 \quad \text{or } 267\tfrac{1}{22}$$

You may wonder why in the world I would spend so much time discussing these problems that end up quite tedious. I mean, you probably felt comfortable working this out in the first few steps.

Examples

The truth is that this little process that you are mastering actually holds the key to one of the slickest secrets of math. That secret is how you can turn fractions into decimals. A quick example: $\frac{2}{5}$

We know that $\frac{2}{5}$ has many other names such as $\frac{6}{15}, \frac{8}{20}, \frac{20}{50}$ and $\frac{26}{65}$, but there is also another one. We can write this problem like:

$5\overline{)2}$ which at first glance doesn't seem to go anywhere.

But remember 2 is the same as 2.0; now we write:

$5\overline{)2.0}$ notice if we put the decimal point in our answer:

.4 $5\overline{)2.0}$ 20	5 goes into 20 about 4 times Multiply 4x5 = 20
.4 $5\overline{)2.0}$ -20 0	Subtract. No remainder

Here we have .4 or $\frac{4}{10}$ as more names for $\frac{2}{5}$.

Try another one. Write $\frac{3}{8}$ as a decimal:

$8\overline{)3.000}$	Add on as many zeros as we might need
.3 $8\overline{)3.000}$ -24 6	8 goes into 30 about 3 times. Multiply 3x8 = 24 Subtract.
.37 $8\overline{)3.000}$ -24 60 -56 4	Bring down the next column. 8 goes into 60 about 7 times. Multiply 7x8 = 56 Subtract.
.375 $8\overline{)3.000}$ -24 60 -56 40 -40 0	Bring down the next column. 8 goes into 40 about 5 times. Multiply 5x8 = 40 Subtract. Finished. We ran out of columns

Thus $\frac{3}{8}$ = .375

How about one that won't stop:

Write $\frac{4}{9}$ as a decimal:

$9\overline{)4.0000}$	Write a few zeros, just to be safe.
$.4$ $9\overline{)4.0000}$ $\underline{-36}$ 4	9 goes into 40 about 4 times. Multiply 4x9 = 36 Subtract.
$.44$ $9\overline{)4.0000}$ $\underline{-36}$ 40 $\underline{-36}$ 4	Bring down the next column. 9 goes into 40 about 4 times. Multiply 4x9 = 36 Subtract.
$.444$ $9\overline{)4.0000}$ $\underline{-36}$ 40 $\underline{-36}$ 40 $\underline{-36}$ 4	Bring down the next column. 9 goes into 40 about 4 times. Multiply 4x9 = 36 Subtract. This could go on forever!

Repeating decimal Thus $\frac{4}{9} = .44444...$ which we simply write by $.\overline{4}$

The bar signifies numbers or patterns that repeat.

There are a few things this little trick allows you to do:

1) You can **divide decimals**. It turns out to be quite nice.

$36.92 \div 7.1 = \frac{36.92}{7.1} = \frac{369.2}{71} = 71\overline{)369.2}$ then you can just do it like we did above. *That is the whole secret.*

Examples

Summary of Decimal Division:

Division of Decimals

1. Set up.
    1. Move decimals
    2. Add zeros
2. Divide into first.
3. Multiply.
4. Subtract.
5. Drop down.
6. Write answer.
    1. Remainder
    2. Decimal

2) You can change any fraction into a decimal.

3) You can leave the world of remainders. (Keep dividing with decimal)

Section 2.4 Exercises

2.1

Create each fraction with a denominator of 24.

1. $\frac{2}{3}$ **2.** $\frac{7}{12}$ **3.** $\frac{40}{48}$

2.2

Add or Subtract. Simplify.

4. $\frac{2}{5}+\frac{2}{7}=$ **5.** $\frac{3}{4}+\frac{7}{9}=$ **6.** $\frac{5}{12}-\frac{7}{8}=$

7. $3\frac{2}{3}-16\frac{7}{9}=$ **8.** $7\frac{5}{7}+6\frac{5}{6}=$ **9.** $2\frac{5}{8}-9\frac{3}{5}=$

Fill out the table.

	Mixed	Improper
10.	$-2\frac{5}{9}$	
11.	$6\frac{4}{7}$	
12.		$\frac{35}{8}$
13.		$\frac{57}{11}$

Find the multiplicative inverse or reciprocal of each number.

2.3

14. $\frac{3}{5}$ **15.** $3\frac{4}{9}$ **16.** $-\frac{5}{12}$ **17.** 7

Divide.

18. $\frac{2}{7}\div\frac{5}{3}=$ **19.** $\frac{3}{4}\div\frac{6}{7}=$ **20.** $\frac{1}{6}\div\frac{4}{9}=$

21. $2\frac{2}{3}\div\frac{3}{10}=$ **22.** $\frac{5}{8}\div 4\frac{1}{2}=$ **23.** $-2\frac{3}{7}\div\frac{5}{7}=$

Change into a decimal.

Example: See examples in section 2.4 for $\frac{4}{9}$

2.4

24. $\frac{2}{5}$ **25.** $\frac{1}{4}$ **26.** $\frac{3}{8}$

27. $\frac{1}{9}$

28. $\frac{7}{8}$

29. $\frac{1}{6}$

Change into a fraction and simplify.

Example
.12

$.12 = 12\ (100^{th}) = \frac{12}{100} \xrightarrow{\text{simplify}} = \frac{3}{25}$

30. .5

31. .7

32. .45

33. .52

34. .75

35. .6

Divide.
Example: See examples in section 2.4.

36. $7\overline{)234}$

37. $5\overline{)135}$

38. $11\overline{)589}$

39. $.04\overline{)56.3}$

40. $.8\overline{)42}$

41. $2.1\overline{)151.2}$

42. $4.2\overline{)45.57}$

43. $.7\overline{)23.48}$

44. $.0005\overline{)36.4}$

Preparation. **Read a little of section 2.5 and then** :

45. If you drive 240 miles on 12 gallons of gas, how many miles per gallon do you get?

46. If you drive 240 miles on 12 gallons of gas, and gas is $4.00 per gallon, how many miles per dollar do you get.

Answers:

1. $\frac{16}{24}$

2. $\frac{14}{24}$

3. $\frac{20}{24}$

4. $\frac{24}{35}$

5. $1\frac{19}{36}$

6. $-\frac{11}{24}$

7. $-13\frac{1}{9}$

8. $14\frac{23}{42}$

9. $-6\frac{39}{40}$

10. $-\frac{23}{9}$

11. $\frac{46}{7}$

12. $4\frac{3}{8}$

13. $2\frac{5}{11}$

14. $\frac{5}{3}$

15. $\frac{9}{31}$

16. $-\frac{12}{5}$

17. $\frac{1}{7}$

18. $\frac{6}{35}$

19. $\frac{7}{8}$

20. $\frac{3}{8}$

21. $8\frac{8}{9}$

22. $\frac{5}{36}$

23. $-3\frac{2}{5}$

24. .4

25. .25

26. .375

27. $.\overline{1}$

28. .875

29. $.1\overline{6}$

30. $\frac{1}{2}$

31. $\frac{7}{10}$

32. $\frac{9}{20}$

33. $\frac{13}{25}$

34. $\frac{3}{4}$

35. $\frac{3}{5}$

36. $33\frac{3}{7}$ or $33.\overline{428571}$ or 33 R3

37. 27

38. $53\frac{6}{11}$ or $53.\overline{54}$ or 53 R6

39. 1407.5

40. 52.5

41. 72

42. 10.85

43. $33.5\overline{428571}$

44. 72,800

45. In class.

46. In class.

Section 2.5

Unit Conversions

Now that you have had a little time to multiply fractions together and simplify them, you may have noticed one of the slickest tricks that we can do with fractions, and that is that we can actually do the simplification before we multiply them. Take for example:

$$\frac{10}{63}\times\frac{21}{55}$$

Now, we can do this the normal way or we can try to notice if there is anything that we will be simplifying out later . . . and do that simplification before we multiply:

Normal method:

$$\frac{10}{63}\times\frac{21}{55}=\frac{210}{3465}$$

and now we try to simplify

$$\frac{210}{3465}=\frac{2\times3\times5\times7}{3\times3\times5\times7\times11}$$

which probably took quite a while to get.

So,

$$\frac{210}{3465}=\frac{2\times3\times5\times7}{3\times3\times5\times7\times11}=\frac{2\times\cancel{3}\times\cancel{5}\times\cancel{7}}{3\times\cancel{3}\times\cancel{5}\times\cancel{7}\times11}=\frac{2}{33}$$

New and improved slick method:

$$\frac{10}{63}\times\frac{21}{55}=$$

and we try to see if anything will cancel ahead of time

$$\frac{\overset{2\times\cancel{5}}{10}}{\underset{3\times\cancel{3}\times\cancel{7}}{63}}\times\frac{\overset{\cancel{3}\times\cancel{7}}{21}}{\underset{\cancel{5}\times11}{55}}=\frac{2}{33}$$

What I was hoping to show is that the same answer was obtained and the same cancelling was done, but if you are able to see it before you multiply, then you will be able to simplify in a much simpler way. Here is another example:

$$\frac{\textcircled{4}}{11}\times\frac{5}{\textcircled{8}}$$

the 4 and the 8 can simplify before we multiply:

$$\frac{\cancel{4}}{11}\times\frac{5}{\cancel{8}_2}=\frac{5}{22}$$

This may seem like just a convenient way to make the problem go a bit quicker, but it does much more than that. It opens the door to a much larger world. Here is an example. If we travelled 180 miles on 12 gallons of gas, then we calculate the mileage by $\frac{180\text{ miles}}{12\text{ gallons}}$ = 15 miles per gallon.

Carrying that example just a bit further, what if gas were $3.2 per gallon? We can actually find how many miles we can drive for one dollar:

$$\frac{180\text{ miles}}{12\ \cancel{\text{gallons}}}\times\frac{1\ \cancel{\text{gallon}}}{3.2\text{ dollars}}=4.7\text{ miles per dollar.}$$

Examples

Another example:

Carpet is on sale for 15 dollars per square yard. How much is that in dollars per square foot (9 ft^2 per yd^2)?

Now, knowing that we will be able to cancel anything on the top with anything that is the same on the bottom we write the multiplication so the the yd^2 will cancel out, leaving us with dollars per ft^2:

$$\frac{15\text{ dollars}}{1\text{ yd}^2} \times \frac{1\text{ yd}^2}{9\text{ ft}^2}$$

Then cancel the yd^2:

$$\frac{15\text{ dollars}}{1\,\cancel{\text{yd}^2}} \times \frac{1\,\cancel{\text{yd}^2}}{9\text{ ft}^2} = \frac{5}{3}\text{ dollars per square foot}$$

= \$1.67 per square foot.

$$\begin{array}{r} 1.66\overline{6} \\ 3\overline{)5.00} \\ \underline{-3} \\ 2\,0 \\ \underline{-18} \\ 20 \\ \underline{-18} \end{array}$$

One more example:

A rope costs \$15 for 8 feet. How much does is cost per inch?

We want to get rid of feet and get inches, so we write the multiplication:

$$\frac{\cancel{15}_5\text{ dollars}}{8\,\cancel{\text{feet}}} \times \frac{1\,\cancel{\text{foot}}}{\cancel{12}_4\text{ inches}} = \frac{5\text{ dollars}}{32\text{ inches}} =$$

\$.156 or 15.6 cents per inch.

$$\begin{array}{r} .1562 \\ 32\overline{)5.00} \\ \underline{-3\,2} \\ 180 \\ \underline{-160} \\ 200 \\ \underline{-192} \\ 80 \end{array}$$

Here are a few numbers that will help you with the conversions:

12 in = 1 foot

16 oz = 1 pound

60 seconds = 1 minute

1000watts = 1 kilowatt

1 yd = 3 ft

60 minutes = 1 hour

1 yd^2 = 9 ft^2

Examples

Section 2.5 Exercises

1.7

Find.

1. $(9.23\text{x}10^{12})\text{x}(2.1\text{x}10^{17})$ **2.** $(4.73\text{x}10^{17})+(2.1\text{x}10^{16})$ **3.** $(8.73\text{x}10^{-7})+(2.1\text{x}10^{-6})$

2.2

Add or Subtract. Simplify.

4. $4\frac{2}{3}-134\frac{7}{9}=$ **5.** $75\frac{7}{12}+16\frac{3}{4}=$ **6.** $21\frac{3}{5}-97\frac{2}{15}=$

Fill out the table.

	Mixed	Improper
7.	$-8\frac{5}{7}$	
8.	$4\frac{9}{13}$	
9.		$\frac{35}{4}$
10.		$\frac{57}{6}$

Find the multiplicative inverse or reciprocal of each number.

2.3

11. $\frac{18}{5}$ **12.** $3\frac{4}{11}$ **13.** $-\frac{17}{5}$ **14.** -19

Divide.

15. $\frac{2}{7}\div\frac{9}{4}=$ **16.** $\frac{9}{4}\div\frac{7}{8}=$ **17.** $\frac{12}{5}\div\frac{8}{15}=$

18. $2\frac{7}{8}\div\frac{1}{10}=$ **19.** $\frac{5}{12}\div4\frac{1}{6}=$ **20.** $12\frac{2}{5}\div2\frac{4}{5}=$

Change into a decimal.

2.4

21. $\frac{17}{5}$ **22.** $\frac{1}{9}$ **23.** $\frac{9}{10}$

24. $\frac{5}{3}$ **25.** $\frac{6}{11}$ **26.** $\frac{1}{6}$

Change into a fraction and simplify.

27. .8

28. .65

29. .57

30. .875

31. .4

32. .44

Divide.

33. $6\overline{)734}$

34. $4\overline{)179}$

35. $11\overline{)393}$

36. $.07\overline{)51.98}$

37. $.8\overline{)423.5}$

38. $2.1\overline{)71.2}$

39. $6.2\overline{)75.64}$

40. $.3\overline{)85.07}$

41. $.0005\overline{)362.4}$

2.5 **Multiply.**

42. $\frac{3\ ft}{1\ yd} \times \frac{1760\ yd}{1\ mile}$

43. $\frac{3\ ft}{1\ yd} \times \frac{12\ in}{1\ ft}$

44. $\frac{2\ cups}{1\ pint} \times \frac{2\ pints}{1\ qt} \times \frac{4\ quarts}{1\ gal}$

Convert the following units.

Example: Dog food cost $7.00 for 20 pounds. How many ounces per dollar?

Solution: $\frac{\text{20 pounds}}{\text{7 dollars}} \times \frac{\text{16 ounces}}{\text{1 pound}} = \frac{\text{320 ounces}}{\text{7 dollars}} = 45.71$ ounces per dollar

45. Cereal cost $4.50 for 2 pounds. How much did it cost per ounce?

46. Fishing line costs $.02 per foot. How much would 200 yards cost?

47. I was able to drive 270 miles on 15 gallons of gas. If gas costs $3.00 per gallon, how many miles can I drive per dollar?

48. If my sprinkler sends out 5 gallons per minute, and if water costs $0.65 per 1000 gallons, how much does watering my lawn cost per hour?

49. Cereal costs $3.50 for 2 pounds. How many ounces per dollar?

Preparation.

50. If you earned $\frac{12}{15}$ on a quiz? What percent did you get?

Answers:

1. $1.9383\text{x}10^{30}$
2. $4.94\text{x}10^{17}$
3. $2.973\text{x}10^{-6}$
4. $-130\frac{1}{9}$
5. $92\frac{1}{3}$
6. $-75\frac{8}{15}$
7. $-\frac{61}{7}$
8. $\frac{61}{13}$
9. $8\frac{3}{4}$
10. $9\frac{1}{2}$
11. $\frac{5}{18}$
12. $\frac{11}{37}$
13. $-\frac{5}{17}$
14. $-\frac{1}{19}$
15. $\frac{8}{63}$
16. $\frac{18}{7}$ or $2\frac{4}{7}$
17. $\frac{9}{2}$
18. $\frac{115}{4}$ or $28\frac{3}{4}$
19. $\frac{1}{10}$
20. $\frac{31}{7}$ or $4\frac{3}{7}$
21. 3.785
22. $.\overline{1}$
23. .9
24. $1.\overline{6}$
25. $.\overline{54}$
26. $.1\overline{6}$
27. $\frac{4}{5}$
28. $\frac{13}{20}$
29. $\frac{57}{100}$
30. $\frac{7}{8}$
31. $\frac{2}{5}$
32. $\frac{11}{25}$
33. $122\frac{1}{3}$ or $122.\overline{3}$
34. $44\frac{3}{4}$ or 44.75
35. $35\frac{8}{11}$ or $35.\overline{72}$
36. 742.57
37. 529.375
38. 33.905
39. 12.2
40. 283.57
41. 724,800
42. $\frac{5280\text{ ft}}{\text{mile}}$
43. $\frac{36\text{ in}}{\text{yd}}$
44. $\frac{16\text{ cups}}{\text{gal}}$
45. \$0.14 per ounce
46. \$12.00
47. 6 miles per dollar
48. \$0.20 per hour
49. 9.14 ounces per dollar
50. In class

Section 2.6
Percents

4) You can make everything into the common language of decimals and therefore percents.

This fourth advantage is what we are going to emphasize now. We motivate this by a question.

In a math class, Joe got 15 out of 21 on a quiz. On the next quiz he got 35 out of 50. Obviously Joe got more questions right on the second quiz, but did he really do better? The question is, "Which one of them is a better score?"

So which is bigger?

$$\frac{15}{21} \text{ or } \frac{35}{50}$$

We have to make them into **like things** to be able to tell.

Option 1: get common denominators

$$\frac{15}{21} \qquad\qquad \frac{35}{50}$$

$$\frac{5}{7} \quad \text{simplify} \quad \frac{7}{10}$$

$$\frac{50}{70} \quad \text{complexify} \quad \frac{49}{70}$$

Option 2: turn them both into percents (calculators are handy)

$$\frac{15}{21} = 71.4\% \quad \text{and} \quad \frac{35}{50} = 70\%$$

Either way, you can tell that the $\frac{15}{21}$ was the better score. The more and more fractions you are working with, the harder and harder it is to get all of them to the same common denominator. Percents start to look nicer and nicer. I know that, having taken many classes and received many grades in school, you have a pretty good idea of how percents work, but let's make sure that we are all on the same page.

Percent can be broken up into two words: "**per**" and "**cent**" meaning per hundred, or in other words, hundredths.

$$\frac{7}{100} = 7\% \qquad \frac{31}{100} = 31\% \qquad \frac{53}{100} = 53\%$$

Notice that each fraction also has a decimal representation:

$$7\% = \frac{7}{100} = .07 \qquad 31\% = \frac{31}{100} = .31 \qquad 53\% = \frac{53}{100} = .53$$

Notice the shortcut from decimal to percents**: move the decimal two places.**

Here is some practice so you can see how it works:

Fraction	Decimal/Number	Percent (rounded)
$\frac{3}{8}$	.375	37.5%
$\frac{7}{10}$	.7	70%
$2\frac{1}{2}$	2.5	250%
$\frac{15}{18}$	$.833\overline{3}$	83.3%
$\frac{5}{11}$	$.45\overline{45}$	45.5%
$8\frac{5}{8}$	8.625	862.5%
$\frac{51}{73}$	.698630137 . . .	69.9%

The most important thing that you should know about percents is that they never stand alone. If I were to call out that I owned 35%, the immediate response is, "35% of what?"

Percents always are a percent of something. For example, sales tax is about 6% or 7% of your purchase. Since this is so common, we need to know how to figure this.

In math terms the word "of" means times.

If you buy $25 worth of food and the sales tax is 7%, then the actual tax is 7% of $25.

.07x25 = $1.75

Another: If a shirt costs $18.25 and is on sale for 20% off then you actually save 20% of 18.25

.20x18.25 = $3.65 is the discount. $14.60 is the final price after discounts.

Last one: 550 people attended a meeting. If 26% of them were driving green cars how many people drove green cars?

26% of 550 =

.26x550 =

143 people driving green cars.

Percents

1. Move decimal 2 places.
   1. Right to %
   2. Left to decimal
2. "OF" means times.

Examples

Section 2.6 Exercises

2.1

Create each fraction with a denominator of 80.

1. $\frac{2}{5}$ **2.** $\frac{5}{8}$ **3.** $\frac{3}{20}$

2.2

Add or Subtract. Simplify.

4. $\frac{1}{2}+\frac{5}{7}=$ **5.** $\frac{5}{12}+\frac{7}{18}=$ **6.** $\frac{2}{9}-\frac{2}{3}=$

7. $5\frac{1}{6}-16\frac{7}{9}=$ **8.** $5\frac{7}{10}+13\frac{1}{4}=$ **9.** $2\frac{4}{15}-9\frac{2}{5}=$

Fill out the table.

	Mixed	Improper
10.	$-3\frac{9}{11}$	
11.		$\frac{33}{12}$

Divide.

2.3

12. $\frac{8}{9}\div\frac{4}{3}=$ **13.** $\frac{7}{8}\div 4\frac{5}{6}=$ **14.** $12\frac{2}{3}\div\frac{4}{6}=$

Change into a decimal.

2.4

15. $\frac{1}{8}$ **16.** $\frac{5}{9}$ **17.** $\frac{7}{10}$

Change into a fraction and simplify.

18. .9 **19.** .38 **20.** .76

Divide.

21. $6\overline{)295}$ **22.** $4\overline{)381}$ **23.** $21\overline{)973}$

24. $6.3\overline{)68.27}$ **25.** $.5\overline{)68.51}$ **26.** $.0004\overline{)362.4}$

2.5

27. I was able to drive 350 miles on 15 gallons of gas. If gas costs $3.10 per gallon, how many miles can I drive per dollar?
28. How much does it cost to run an 800 watt microwave for 15 hours per month if the power company charges 13 cents per kilowatt-hour?
29. I just bought a 24 foot rope for $7.00. How many inches per dollar did I get?
30. Tile costs $2.50 per square foot; how much is that per square yard?

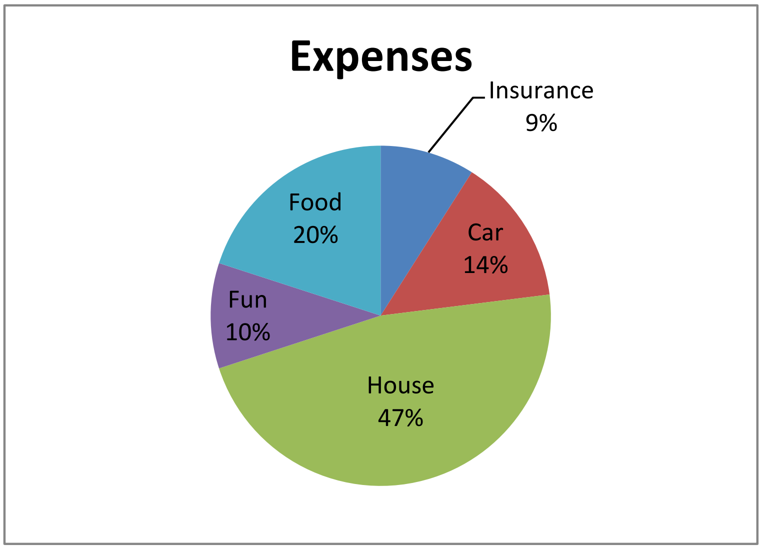
Change into a percent.

2.6

31. $\frac{9}{12}$

32. $\frac{19}{20}$

33. $\frac{15}{45}$



Using the chart, find out how much money was spent if the total budget was $1600.

34. Insurance

35. House

36. Fun

Find the following:

37. Price: $35.20
Tax rate: 6%
Tax:

38. Attendees: 2,300
Percent men: 29%
Men:

39. Students: 12
Number of B's: 9
Percent of B's:

Answers:

1. $\frac{32}{80}$

2. $\frac{50}{80}$

3. $\frac{12}{80}$

4. $1\frac{3}{14}$ or $\frac{17}{14}$

5. $\frac{29}{36}$

6. $-\frac{4}{9}$

7. $-11\frac{11}{18}$

8. $18\frac{19}{20}$

9. $-7\frac{2}{15}$

10. $-\frac{42}{11}$

11. $2\frac{3}{4}$

12. $\frac{2}{3}$

13. $\frac{21}{116}$

14. 19

15. .125

16. $.\overline{5}$

17. .7

18. $\frac{9}{10}$

19. $\frac{19}{50}$

20. $\frac{19}{25}$

21. $49\frac{1}{6}$ or $49.1\overline{6}$

22. $95\frac{1}{4}$ or 95.25

23. $46\frac{1}{3}$ or $46.\overline{3}$

24. 10.8365 . . .

25. 258.34

26. 906,000

27. 7.53 miles per dollar

28. \$1.56

29. 41.14 inches per dollar

30. \$22.50 dollars per square yard

31. 75%

32. 95%

33. 33.3%

34. \$144

35. \$752

36. \$160

37. \$2.11

38. 667

39. 75%

Section 2.7

Order of Operations

Order of Operations

The last small note to finalize all your abilities in arithmetic is to make sure you know what you need to do when you have multiple operations going on at the same time. For example,

2 + 3 x 4 – 5

If you were to read that from left to right you would first add the 2 and the 3 to get 5 and then multiply by 4 to get 20.

Unfortunately, that doesn't jive with what we have learned about what multiplication is. Remember that multiplication is a shorthand way of writing repeated addition. Technically we have:

2 + 3 x 4 – 5 =
2 + 4 + 4 + 4 – 5 = 9.

Ahh, now there is the right answer. It looks like we need to take care of the multiplication as a group, before we can involve it in other computations. **Multiplication is done before addition and subtraction**.

Here is another one:

4 x 3^2 – 7 x 2 + 4

Now remember that exponents are shorthand for a bunch of multiplication that is hidden, so we need to take care of that even before we do multiplication:

4 x 3^2 – 7 x 2 + 4 = Take care of exponents
4 x 9 – 7 x 2 + 4 = Take care of multiplication
36 – 14 + 4 = Add/Sub left to right.
22 + 4 = 26.

Now division can always be written as multiplication of the reciprocal, so make sure you do division before addition and subtraction as well.

Look at that. We have established an order which the operations always follow, and we need to know it if we are to get the answers that the problem is looking for:

1st – Exponents
2nd – Multiplication and Division (glues numbers together)
3rd – Addition and Subtraction (left to right)

Parentheses can change everything. We put parentheses when we intend on grouping (or gluing) numbers together manually. Though they all have the same numbers and operations, see the difference between these:

$$\boxed{\begin{aligned}&2-3\times 6^2 \div 2 = \\ &2-3\times 36 \div 2 = \\ &2-54=-52\end{aligned}}$$

$$\boxed{\begin{aligned}&2-(3\times 6)^2 \div 2 = \\ &2-18^2 \div 2 = \\ &2-324 \div 2 = \\ &2-162=-160\end{aligned}}$$

$$\boxed{\begin{aligned}&(2-3)\times 6^2 \div 2 = \\ &-1\times 36 \div 2 = \\ &-36 \div 2 = -18\end{aligned}}$$

$$\boxed{\begin{aligned}&(2-3\times 6)^2 \div 2 = \\ &(2-18)^2 \div 2 = \\ &(-16)^2 \div 2 = \\ &256 \div 2 = 128\end{aligned}}$$

Section 2.7 Exercises

Estimate the following.

1.2

1. .0000457 x .08395

2. 56,327 x 71,325,000

3. Round 7.85649 to the nearest thousandth.

Add.

1.3

4. 72,371
+ 389.4

5. 653.3
+ 45.7

6. $71\frac{19}{29} + 4\frac{13}{29} =$

Subtract.

1.4

7. Temp: -25.7° F
Change: 130.4° warmer
Final:

8. -8 – (-5) =

9. $13\frac{2}{5} - 18\frac{2}{5} =$

Multiply.

1.5

10. Cost: $135.20
Quantity: 7
Total:

11. 724x73 =

12. $\frac{3}{5} \times \frac{10}{13} =$

Convert into scientific notation.

1.7

13. 52,000

14. $190\text{x}10^{17}$

15. .000494

Convert into decimal notation.

16. $3.48\text{x}10^{7}$

17. $7.3\text{x}10^{10}$

18. $3.84\text{x}10^{-13}$

Find.

19. $(5.2\text{x}10^{-10})+(6.7\text{x}10^{-9})$

20. $(4.23\text{x}10^{10}) - (5.7\text{x}10^{9})$

21. $(4\text{x}10^{8})\text{x}(1.8\text{x}10^{10})$

Assignment 2.7

2.2

Add or Subtract. Simplify.

22. $\frac{2}{5}+\frac{5}{8}=$ **23.** $7\frac{7}{12}+16\frac{1}{4}=$ **24.** $\frac{11}{12}-\frac{7}{8}=$

Fill out the table.

	Mixed	Improper
25.	$-3\frac{5}{7}$	
26.		$\frac{15}{4}$

Divide.

2.3

27. $2\frac{7}{8}\div\frac{13}{10}=$ **28.** $\frac{3}{8}\div 4\frac{1}{6}=$ **29.** $12\frac{2}{5}\div\frac{4}{5}=$

Change into a decimal.

2.4

30. $\frac{5}{9}$ **31.** $\frac{7}{11}$ **32.** $\frac{5}{6}$

Change into a fraction and simplify.

33. .125 **34.** .8 **35.** .48

Divide.

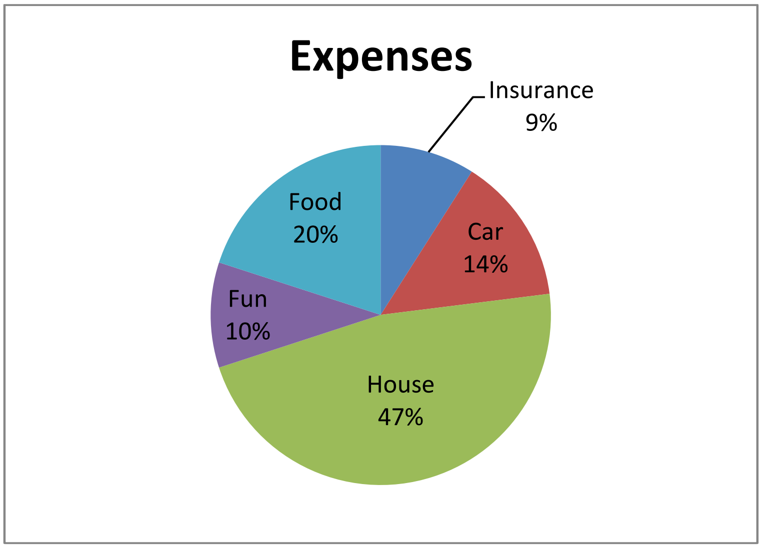
2.5

36. $6.2\overline{)44.64}$ **37.** $.3\overline{)25.47}$ **38.** $.0008\overline{)362.4}$

Change into a percent.

2.6

39. $\frac{9}{12}$ **40.** $\frac{19}{20}$ **41.** $\frac{15}{45}$



Using the chart, find out how much money was spent if the total budget was $3600.

42. Insurance

43. House

44. Fun

Find the following:

45. Price: $55.20
Tax rate: 6%
Tax:

46. Attendees: 1,400
Percent men: 39%
Men:

47. Students: 20
Number of B's: 9
Percent of B's:

Find.

2.7

48. $10-4\div 2+9$

49. $2^3\times 5\div 4-7\times 5$

50. $6\times(4-3)^4-10\div 2$

Answers:

1. .000004

2. 4,200,000,000,000

3. 7.856

4. 72,760.4

5. 699

6. $76\frac{3}{29}$

7. 104.7° F

8. -3

9. -5

10. $946.40

11. 52,852

12. $\frac{6}{13}$

13. $5.2\text{x}10^{4}$

14. $1.9\text{x}10^{19}$

15. $4.94\text{x}10^{-4}$

16. 34,800,000

17. 73,000,000,000

18. .000000000000384

19. $7.22\text{x}10^{-9}$

20. $3.66\text{x}10^{10}$

21. $7.2\text{x}10^{18}$

22. $1\frac{1}{40}$

23. $23\frac{5}{6}$

24. $\frac{1}{24}$

25. $-\frac{26}{7}$

26. $3\frac{3}{4}$

27. $\frac{115}{52}$ or $2\frac{11}{52}$

28. $\frac{9}{100}$

29. $\frac{31}{2}$

30. $.\overline{5}$

31. $.\overline{63}$

32. $.8\overline{3}$

33. $\frac{1}{8}$

34. $\frac{4}{5}$

35. $\frac{12}{25}$

36. 7.2

37. 63.9

38. 453,000

39. 75%

40. 95%

41. $33.\overline{3}\%$

42. $324

43. $1,692

44. $360

45. $3.31

46. 546

47. 45%

48. 17

49. -25

50. 1

Summary of Fractions – Arithmetic Chapter 1

Addition/Subtraction of Fractions

1. Set it up. Subtract with bigger on top
2. Get Common Denominators.
    1. Observation
    2. Multiply the denominators
    3. Prime factorization
3. Add/Subtract numerators.
4. Carry or Borrow by Denominator.
5. Strongest number wins. (+ or -).
6. Simplify.
    1. Observation
    2. Prime factorization

Multiplication of fractions

1. No common denominator
2. Multiply numerators.
3. Multiply denominators.
4. Simplify.
    1. Observation
    2. Prime factorization

Division of fractions

1. Multiply by reciprocal.

Division of Decimals

1. Set up.
    1. Move decimals
    2. Add zeros
2. Divide into first.
3. Multiply.
4. Subtract.
5. Drop down.
6. Write answer.
    1. Remainder
    2. Decimal

Fraction Review Assignment

1. Create a visual chart on one side of a 8.5” X 11” paper with all of the rules used to add, subtract, multiply, divide and convert fractions to decimals and percents and how to deal with negatives.

Arrange each set of fractions, from greatest to least:

2. $\frac{5}{9}, \frac{3}{4}, \frac{1}{2}, \frac{5}{6}, \frac{2}{3}$ _______, _______, _______, _______, _______

3. $\frac{11}{100}, \frac{9}{100}, \frac{101}{1000}, \frac{111}{1000}, \frac{1}{10}$ _______, _______, _______, _______, _______

4. $-\frac{2}{3}, -\frac{1}{6}, -\frac{3}{10}, -\frac{1}{4}, -\frac{2}{5}$ _______, _______, _______, _______, _______

Convert the following into a fraction:

5. 0.025 ____________

6. 35.7% ____________

7. 0.16 ____________

8. –110% ____________

9. 98.76% ____________

Add and simplify the following:

10. $4\frac{2}{7} + \frac{5}{7}$ ____________

11. $5\frac{3}{8} + 1\frac{1}{6}$ ____________

12. $14\frac{1}{4} + 2\frac{11}{20}$ ____________

Subtract and simplify the following:

13. $4\frac{2}{7} - \frac{5}{7}$ ____________

14. $5\frac{3}{8} - 1\frac{1}{6}$ ____________

15. $-14\frac{1}{4} - 2\frac{11}{20}$ ____________

Multiply and simplify the following:

16. $3\frac{4}{5}$ x $1\frac{2}{3}$ ____________

17. $-\frac{1}{4}$ x $\frac{7}{10}$ ____________

18. -60 x $-\frac{5}{4}$ ____________

Divide and simplify the following:

19. $3\frac{4}{5} \div 1\frac{2}{3}$ ____________

20. $-\frac{1}{4} \div \frac{7}{10}$ ____________

Summary of Decimals – Arithmetic Chapter 1

Addition/Subtraction of Decimals

1. Set it up.
    1. Line up decimals
    2. Subtract with bigger on top
2. Add/Subtract columns.
3. Carry or Borrow by 10’s.
4. Strongest number wins (+ or -).

Addition/Subtraction of Scientific Notation

1. Set it up.
    1. Get same exponent (left-up)
    2. Subtract with bigger on top
2. Add/Subtract columns.
3. Carry or Borrow by 10’s.
4. Strongest number wins.
5. Put into scientific notation (left-up).

Multiplication of Decimals

1. Multiply each place value.
2. Carry by 10’s.
3. Add.
4. Right Size
    1. Add up zeros or decimals
    2. Negatives

Multiplication of Scientific Notation

1. Multiply decimals.
2. Add exponents.
3. Put into scientific notation (left-up).

Division of Decimals

1. Set up.
   1. Move decimals
   2. Add zeros
2. Divide into first.
3. Multiply.
4. Subtract.
5. Drop down.
6. Write answer.
   1. Remainder
   2. Decimal

Percents

1. Move decimals two places
2. “OF” means times

Rounding

1. Find the place of rounding.
2. Examine previous place.
3. 5 or bigger goes up.
4. 4 or less stays same.
5. All following values are zero.

Estimation

1. Round to the highest value.
2. Do the easy problem.

Decimal Review Assignment

1. Create a visual chart on one side of a 8.5” X 11” paper with all of the rules used to add, subtract, multiply, divide and convert decimals into percents and fractions, and how to deal with negatives.

Arrange each set of decimals, from least to greatest:

2. 0.4, 0.44, 0.04, 0.404, 0.044 _______, _______, _______, _______, _______

3. 0.11, 0.13, 0.12, 0.1111, 0.123 _______, _______, _______, _______, _______

4. –0.6, –0.67, –0.606, –0.66, –0.667 _______, _______, _______, _______, _______

Convert the following into a decimal:

5. 1/16 ____________

6. $35.\overline{7}\%$ ____________

7. 4/9 ____________

8. –110% ____________

9. 67.89% ____________

Add the following:

10. 909 + 22.473 ____________

11. –67.89 + 123.4 ____________

12. 50.05 + 173.95 ____________

Subtract the following:

13. 224.73 – 22.473 ____________

14. –37 – 98.6 ____________

15. 195.76 – 16.48 ____________

Multiply the following:

16. 360 x –3.21 ____________

17. 8.5 x 6.56 ____________

18. –34.5 x –76.5 ____________

Divide the following:

19. 1,086.42 ÷ 30 ____________

20. 369.6 ÷ –7 ____________

Chapter 1 and 2 Review Exercises

Estimate the product (round to the greatest value, then multiply).

1.2

1. 2,589,000x59.34

2. .005608x.07816

3. 3.847x2,564

Add.

1.3

4. 36,841
$\underline{+\ 249.7}$

5. 723.3
$\underline{+\ 39.7}$

6. $16\frac{9}{14}+5\frac{13}{14}=$

Subtract.

1.4

7. Temp: -35.5° F
Change: 13.4° warmer
Final:

8. -8 – (-11) =

9. $13\frac{4}{7}-1\frac{6}{7}=$

Multiply.

1.5

10. Cost: $35.20
Quantity: 17
Total:

11. 369x(-23) =

12. $\frac{4}{5}\times\frac{11}{12}=$

Find.

1.6

13. $7^4=$

14. $18^2=$

15. 10^{14}

Convert into scientific notation.

1.7

16. 381,000

17. $.0075\text{x}10^{17}$

18. .00328

Convert into decimal notation.

19. $3.51\text{x}10^{-7}$

20. $7.3\text{x}10^{9}$

21. $7.91\text{x}10^{13}$

Find.

22. $(7.3\text{x}10^{-10})+(8.4\text{x}10^{-9})$ **23.** $(3.1\text{x}10^{9})$ - $(4.9\text{x}10^{8})$ **24.** $(4.5\text{x}10^{8})\text{x}(8\text{x}10^{15})$

Simplify.

2.7

25. $2^5+4-(8+3^2)$ **26.** $(7-4)^2-12+6\div 3$ **27.** $(2^4+8)\div 2-6$

2.2

Add or Subtract. Simplify.

28. $\frac{3}{2}+\frac{5}{9}=$ **29.** $\frac{11}{12}+\frac{5}{14}=$ **30.** $\frac{5}{18}-\frac{5}{6}=$

31. $15\frac{1}{6}-6\frac{7}{9}=$ **32.** $5\frac{9}{10}+13\frac{1}{8}=$ **33.** $12\frac{4}{9}-9\frac{2}{14}=$

Fill out the table.

	Mixed	Improper
34.	$-3\frac{5}{7}$	
35.		$\frac{59}{6}$

Divide.

2.3

36. $\dfrac{\frac{8}{9}}{\frac{5}{18}}$ **37.** $\frac{8}{9}\div 4\frac{2}{3}=$ **38.** $7\frac{3}{4}\div\left(-\frac{4}{5}\right)=$

Change into a decimal.

2.4

39. $\frac{5}{12}$ **40.** $\frac{7}{9}$ **41.** $\frac{2}{7}$

Change into a fraction and simplify.

42. .3 **43.** .055 **44.** .375

Divide.

45. $7\overline{)485}$ **46.** $3\overline{)781}$ **47.** $43\overline{)673}$

48. $.5\overline{)47.31}$

49. $.0004\overline{)562.4}$

2.5

50. A dishwasher uses about 1400 watts of power. If the power company charges 9 cents per kilowatt-hour, how much does it cost to run a dishwasher for 16 hours in the month?

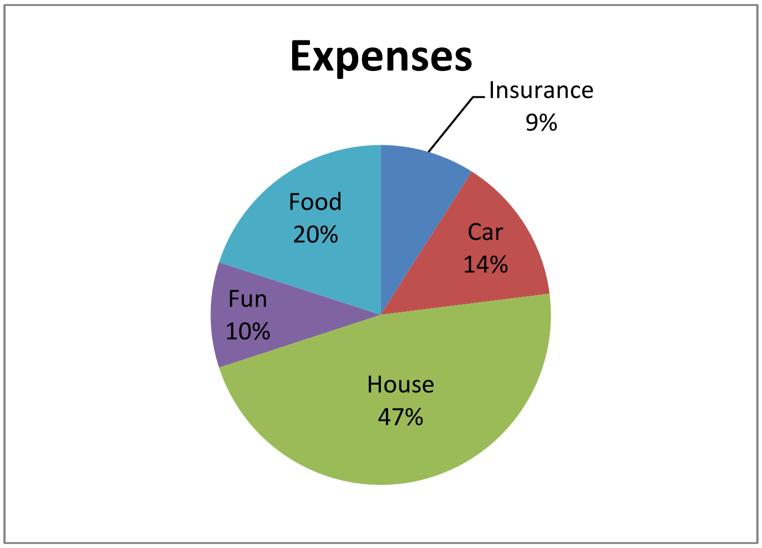
Change into a percent.

2.6

51. $\frac{24}{25}$

52. $\frac{36}{40}$

53. $\frac{17}{50}$



Using the chart, find out how much money was spent if the total budget was $3200.

54. Car

55. House

56. Food

Find the following:

57. Price: $45.20
Tax rate: 7%
Tax:

58. Attendees: 239
Percent men: 29%
Men:

59. Students: 15
Number of A's: 14
Percent of A's:

1.2

60. Round to the nearest ten:
583.872

Answers:

1. 180,000,000
2. .00048
3. 12,000
4. 37,090.7
5. 763
6. $22\frac{4}{7}$
7. -22.1° F
8. 3
9. $11\frac{5}{7}$
10. $598.40
11. -8,487
12. $\frac{11}{15}$
13. 2401
14. 324
15. 100,000,000,000,000
16. $3.81\text{x}10^{5}$
17. $7.5\text{x}10^{14}$
18. $3.28\text{x}10^{-3}$
19. .000000351
20. 7,300,000,000
21. 79,100,000,000,000
22. $9.13\text{x}10^{-9}$
23. $2.61\text{x}10^{9}$
24. $3.6\text{x}10^{24}$
25. 19
26. -1
27. 6
28. $2\frac{1}{18}$
29. $1\frac{23}{84}$
30. $-\frac{5}{9}$
31. $8\frac{7}{18}$
32. $19\frac{1}{40}$
33. $3\frac{19}{63}$
34. $-\frac{26}{7}$
35. $9\frac{5}{6}$
36. $3\frac{1}{5}$ or $\frac{16}{5}$
37. $\frac{4}{21}$
38. $-9\frac{11}{16}$ or $-\frac{155}{16}$
39. $.41\overline{6}$
40. $.\overline{7}$
41. $.\overline{285714}$
42. $\frac{3}{10}$
43. $\frac{11}{200}$
44. $\frac{3}{8}$
45. $69\frac{2}{7}$ or $69.\overline{285714}$
46. $260\frac{1}{3}$ or $260.\overline{3}$
47. $15\frac{28}{43}$ or 15.65116…
48. 94.62
49. 1,406,000
50. $2.02
51. 96%
52. 90%
53. 34%
54. $448
55. $1504
56. $640
57. $3.16
58. 69
59. 93.3%
60. 580

Calculator Usage Assignment

On this assignment, you should **use your calculator**. Become familiar with it. It is now your friend!

1.2

Estimate the product (round to the greatest value; then multiply).

1. 75,800x49.34

2. .004208x.06916

3. 4.447x7,164

Add.

1.3

4. $\begin{array}{r} 37{,}291 \\ \underline{+\ 348.23} \end{array}$

5. $\begin{array}{r} 5.871 \\ \underline{+\ 39.7} \end{array}$

6. $17\frac{9}{23}+5\frac{5}{23}=$

1.4

Subtract.

7. Temp: 85.3° F
Change: 130.4° colder
Final:

8. -5 –3 =

9. $23\frac{4}{11}-15\frac{8}{11}=$

1.5

Multiply.

10. Cost: $38.40
Quantity: 27
Total:

11. 441x29 =

12. $-\frac{2}{5}\times\frac{16}{11}=$

Find.

1.6

13. $3^5=$

14. $27^2=$

15. 10^7

1.7

Convert into scientific notation.

16. .00004823

17. $.538\text{x}10^{18}$

18. $.0000007\text{x}10^{5}$

Find.

2.7

19. $2+7\times2^2-5$

20. $8+(5-3)^3\div4+7$

21. $16\div4^2+15\div(3+2)$

Chapter 1 and 2 Review Assignment
with Calculator

1.7

Find.

22. $(1.38\text{x}10^{9})+(8.4\text{x}10^{7})$ **23.** $(4.3\text{x}10^{9})$ - $(4.9\text{x}10^{8})$ **24.** $(4.5\text{x}10^{-8})\text{x}(7\text{x}10^{15})$

2.1

Create each fraction with a denominator of 18.

25. $\frac{1}{3}$ **26.** $\frac{5}{9}$ **27.** $\frac{5}{6}$

2.2

Add or Subtract. Simplify.

28. $\frac{3}{4}+\frac{4}{9}=$ **29.** $\frac{5}{8}+\frac{7}{10}=$ **30.** $\frac{8}{15}-\frac{7}{9}=$

31. $14\frac{1}{8}-7\frac{4}{9}=$ **32.** $5\frac{9}{10}+19\frac{1}{8}=$ **33.** $4\frac{5}{8}-5\frac{3}{16}=$

Fill out the table.

	Mixed	Improper
34.	$7\frac{4}{11}$	
35.		$-\frac{5}{2}$

Divide.

2.3

36. $\frac{\frac{11}{12}}{\frac{7}{18}}$ **37.** $\frac{5}{6}\div 4\frac{1}{2}=$ **38.** $7\frac{5}{8}\div\frac{3}{8}=$

Change into a decimal.

2.4

39. $\frac{7}{11}$ **40.** $\frac{3}{5}$ **41.** $\frac{2}{9}$

Change into a fraction and simplify.

42. .07 **43.** .44 **44.** .625

Divide.

45. $7\overline{)343}$ **46.** $6\overline{)79}$ **47.** $57\overline{)6273}$

48. $.5\overline{)4.731}$

49. $.004\overline{)967.4}$

2.5

50. Change 60 miles per hour into feet per second. (5280 feet = 1 mile)

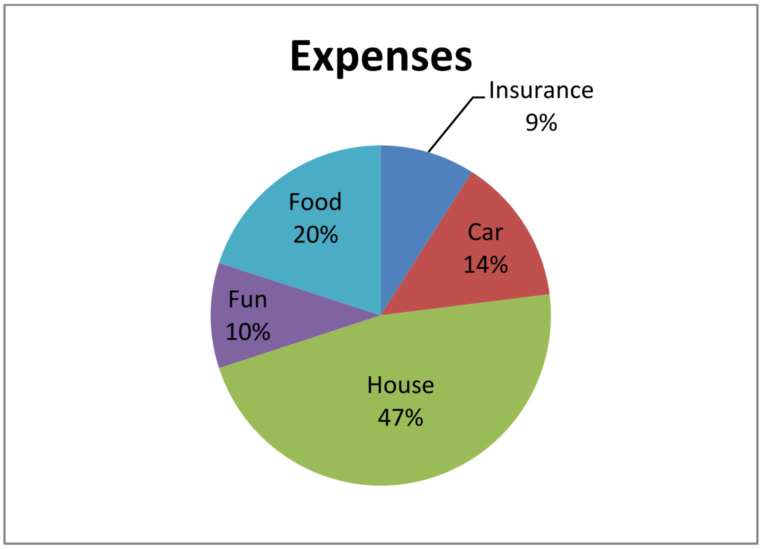
Change into a percent.

2.6

51. $\frac{28}{30}$

52. $\frac{41}{50}$

53. $\frac{37}{100}$



Using the chart, find out how much money was spent if the total budget was $2400.

54. Fun

55. Insurance

56. Food

Find the following:

57. Price: $380.50
Tax rate: 7%
Tax:

58. Attendees: 48
Percent kids: 25%
Kids:

59. Students: 30
Number of A's: 24
Percent of A's:

1.2

60. Round to the nearest hundredth:
583.872

Answers:

1. 4,000,000
2. .00028
3. 28,000
4. 37,639.23
5. 45.571
6. $22\frac{14}{23}$
7. -45.1° F
8. -8
9. $7\frac{7}{11}$
10. $1,036.80
11. 12,789
12. $-\frac{32}{55}$
13. 243
14. 729
15. 10,000,000
16. $4.823\text{x}10^{-5}$
17. $5.38\text{x}10^{17}$
18. $7\text{x}10^{-7}$
19. 25
20. 17
21. 4
22. $1.464\text{x}10^{9}$
23. $3.81\text{x}10^{9}$
24. $3.15\text{x}10^{8}$
25. $\frac{6}{18}$
26. $\frac{10}{18}$
27. $\frac{15}{18}$
28. $1\frac{7}{36}$
29. $1\frac{13}{40}$
30. $-\frac{11}{45}$
31. $6\frac{49}{72}$
32. $25\frac{1}{40}$
33. $-\frac{9}{16}$
34. $\frac{81}{11}$
35. $-2\frac{1}{2}$
36. $2\frac{5}{14}$ or $\frac{33}{14}$
37. $\frac{5}{27}$
38. $20\frac{1}{3}$ or $\frac{61}{3}$
39. $.\overline{63}$
40. .6
41. $.\overline{2}$
42. $\frac{7}{100}$
43. $\frac{11}{25}$
44. $\frac{5}{8}$
45. 49
46. $13\frac{1}{6}$ or $13.1\overline{6}$
47. 110.0526…
48. 9.462
49. 241,850
50. 88 feet per second
51. 93.3%
52. 82%
53. 37%
54. $240
55. $216
56. $480
57. $26.64
58. 12
59. 80%
60. 583.87

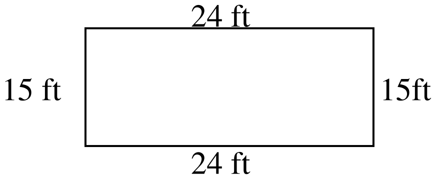
Chapter 3:

VARIABLES

Overview

Section 3.1
Variables

You have the ability to add, subtract, multiply and divide any numbers that come your way. Just to make sure you have it, let's see you tell me how far you would be walking around this rectangle.



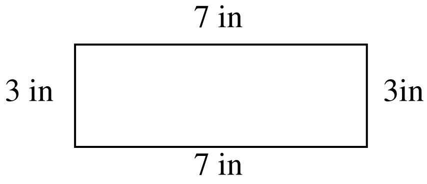
Not so bad. It appears that to get all the way around it, we simply add up the numbers on each side until we get all the way around.

24+15+24+15 = 78.

So if you walked around a 24ft X 15ft rectangle, you would have completed a walk of 78 ft.

Again:

What if you walked around this rectangle?

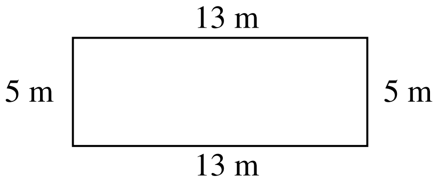


Now the addition of all of the sides is:

7+3+7+3 = 20

20 inches

Again:
What if you walked around this rectangle:



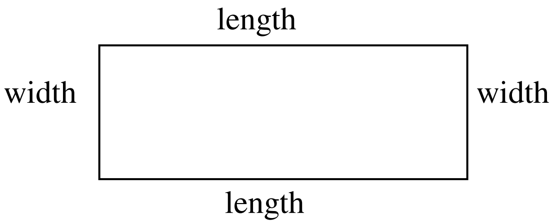
Now the distance all the way around is:

13+5+13+5 = 36

36 meters.

With the three of these examples can you see that we did the exact same thing, only with different numbers. I bet we could come up with the **pattern** for how we would do this all of the time.

Well, first of all, we just pick general terms for the sides of the rectangle:



Then we get something like this:

Distance around the rectangle = length + width + length + width

That seems kind of tedious to write out. Let's try and use some abbreviations. First, there is a handy word, "**perimeter**" that means "around measure". That is a little shorter:

Perimeter = length + width + length + width

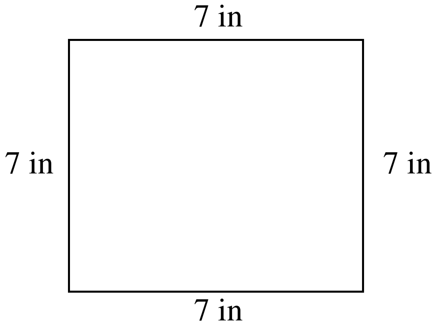
Let's go a bit more with just using the first letters of the words:

$P = l + w + l + w$

Definition of variable.

Ahhh, now that is something that I can get used to. Notice now how **each letter stands for a number** that we could use. The number can change from time to time. This pattern that we have created to describe all cases is called a formula. **The letters used in a formula are called variables**. **The variables, being letters, actually represent numbers, but the numbers can change from time to time, or vary. Thus are they called variables.**

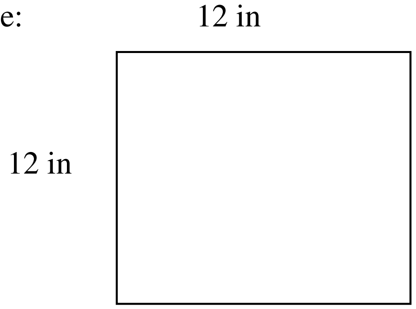
Let's make another formula. This time we will try with a square.
Find the perimeter of (distance around) this square:



Perimeter = 7+7+7+7 = 4x7 = 28

28 inches

Another square:



Perimeter = 12+12+12+12 = 4x12 = 48
48 inches

In this example we don't have a particular length or a width, because all of the sides are the same length. So, the general formula would look like:

Perimeter = 4 x side length or in other words

P = 4xs
where s is the abbreviation for "side length"

Uh oh! I see a problem. Up until now, we have used the letter x for multiplication. If we are going to have variables and formulas and math with letters in them, then **we need something else to mean multiplication**.

Throughout the years people have come up with two other ways of writing multiplication.

They are
1) to put two things next to each other and
2) using a dot instead of an x.

Here are how they are used:

x	**next to**	**dot**
4x3 = 12	(4)(3) = 12	$4\cdot 3 = 12$
7x8 = 56	7(8) = 56	$7\cdot 8 = 56$
2x2x3x5 = 60	(2)(2)(3)(5) = 60	$2\cdot 2\cdot 3\cdot 5=60$
not used	P = 4s	$P = 4\cdot s$
$3x10^5$	$3(10^5)$	$3\cdot 10^5$
not used	12b	$12\cdot b$

Section 3.1 Exercises

Add or Subtract. Simplify.

2.2

1. $5\frac{1}{3}-13\frac{7}{8}=$

2. $7\frac{7}{12}+1\frac{1}{5}=$

3. $21\frac{4}{10}-9\frac{2}{15}=$

Fill out the table.

	Mixed	Improper
4.	$-7\frac{5}{8}$	
5.		$\frac{39}{11}$

Divide.

2.3

6. $2\frac{5}{7}\div\frac{13}{10}=$

7. $12\frac{3}{7}\div4\frac{1}{6}=$

8. $8\frac{2}{5}\div\frac{4}{5}=$

Change into a decimal.

2.4

9. $\frac{7}{9}$

10. $\frac{1}{8}$

11. $\frac{343}{100}$

Change into a fraction and simplify.

12. .95

13. .64

14. .375

Divide.

15. $9\overline{)437}$

16. $6\overline{)235}$

17. $26\overline{)873}$

18. $8.2\overline{)45.74}$

19. $.6\overline{)2.547}$

20. $.0008\overline{)3.624765}$

2.5

21. How many miles per dollar do I get if I get 37 miles per gallon of gas at a price of $3.20 per gallon?

22. What is the price per ounce of flour purchased at $7.00 for 10 pounds?

Find the following:

2.6

23. Price: $580.50
Tax rate: 7%
Tax:
Total Price:

24. Price: $27.00
Percent off: 35%
Amount saved:
Final Price

25. Attempted: 48
Made: 30
Percent of shots made:

Find the following:

3.1

Example:
$7(2) - 3(7) + 5 \cdot 2 \cdot m$

$14 - 21 + 10m$
$\boxed{-7 + 10m}$

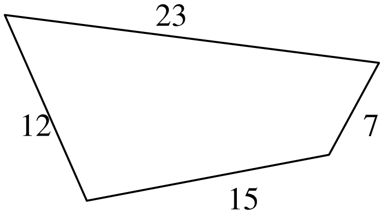
26. $7 \cdot 3 + 2(4)$ **27.** $5 \cdot 2 \cdot 2 \cdot 3$ **28.** $5(3-9) - 7 \cdot 4$

29. $3\frac{1}{8} \cdot \frac{7}{8}$ **30.** $5 + 3 \cdot \frac{5}{8} + 4$ **31.** $5[8-(-4)] + 8 \cdot 2$

32. $24(7.2)$ **33.** $3 \cdot 3 \cdot 3 \cdot x$ **34.** $2(3)+5(7)+8 \cdot 4$

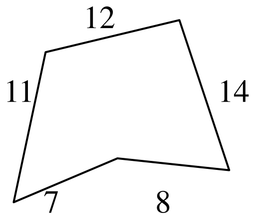
Find the perimeter of the following shapes:

Example:

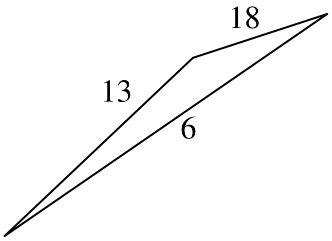


12+23+7+15 = 57
All sides added up = Perimeter is 57

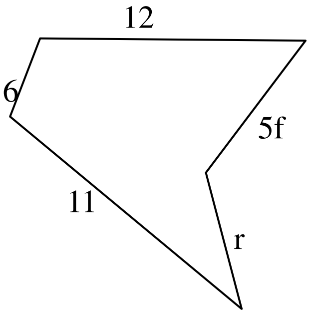
35.



36.



37.



Preparation:

38. What is $6 \cdot x$ if x = 7?

Answers:
1. $-8\frac{13}{24}$
2. $8\frac{47}{60}$
3. $12\frac{4}{15}$
4. $-\frac{61}{8}$
5. $3\frac{6}{11}$
6. $\frac{190}{91}$ or $2\frac{8}{91}$
7. $\frac{522}{175}$ or $2\frac{172}{175}$
8. $\frac{21}{2}$ or $10\frac{1}{2}$
9. $.\overline{7}$
10. .125
11. 3.43
12. $\frac{19}{20}$
13. $\frac{16}{25}$
14. $\frac{3}{8}$
15. $48\frac{5}{9}$ or $48.\overline{5}$
16. $39.1\overline{6}$ or $39\frac{1}{6}$
17. 33.5769…
18. 5.5780…
19. 2.245
20. 4,530.956…
21. 11.56 miles per dollar
22. $.044 per ounce
23. $40.64 and $621.14
24. $9.45 and $17.55
25. 62.5%
26. 29
27. 60
28. -58
29. $2\frac{47}{64}$ or $\frac{175}{64}$
30. $10\frac{7}{8}$
31. 76
32. 172.8
33. 27x
34. 73
35. 52
36. 37
37. 29 +r + 5f
38. In class.

Section 3.2

Substitution

From now on in your math career, the most often way to write multiplication will be by putting two numbers (or variables) next to each other. In a way, this is the most natural thing to do. Back in the good old days, we used to add like things together:

1apple + 1apple = 2apples.

Notice that the 2 is actually doing multiplication. Let's practice using some variables and sticking in numbers for them.

Find 7b, if b = 3.

$7b =$

$7(3) = 21$

Find 7b if b = 9.

$7b =$

$7(9) = 63$

Find 7b if b = 13.

$7b =$

$7(13) = 91$

That's it. When we know what a variable is, all we have to do is stick it in.
Here are some other examples:

Evaluate 5t + s if t = 3 and s = 36.

$5t + s =$

$5(3) + 36 =$

$15 + 36 =$

51

Evaluate 6x – 2y if x = 9 and y = 11.

$6x - 2y =$

$6(9) - 2(11) =$

$54 - 22 =$

32

Section 3.2 Exercises

Add or Subtract. Simplify.

2.2

1. $5\frac{3}{4}-23\frac{7}{8}=$

2. $13\frac{3}{10}+1\frac{7}{16}=$

3. $137\frac{4}{10}-9\frac{5}{8}=$

Divide.

2.4

4. $.82\overline{)45740}$

5. $.6\overline{)7.647}$

6. $700\overline{)3561.5}$

2.5

7. If I get 28 miles per gallon, how much is the cost of driving in dollars per mile if gas is $3.00 per gallon?

8. Use a calculator. If a bicycle is traveling at 30 feet per second, how many miles per hour is it going?

Find the following:

2.6

9. Price: $430.50
Tax rate: 11%
Tax:
Total Price:

10. Price: $951.00
Percent off: 35%
Amount saved:
Final Price:

11. Attempted: 150
Made: 137
Percent of shots made:

Find the following:

3.1

12. $7\cdot 6-2(9)$

13. $5\cdot t\cdot 2\cdot 4$

14. $5(3-9)-7\cdot 4$

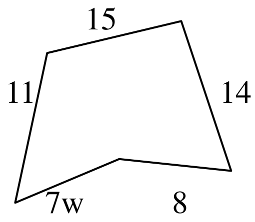
15. $25(9.2)$

16. $3\cdot 3\cdot 3\cdot 2$

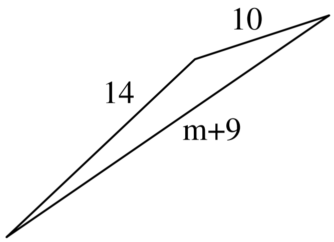
17. $2(3)+5(4)+8\cdot \mathrm{m}$

Find the perimeter of the following shapes:

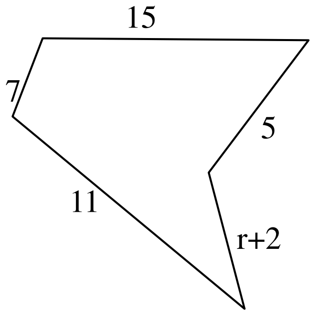
18.



19.



20.



Find the following when x = 5, a = 7, b = 15, and t = 4.

3.2

Example: $4x - b + \frac{20}{t}$

$4(5) - 15 + \frac{20}{4}$

$20 - 15 + 5$

$\boxed{10}$

21. $3x - 7$

22. $2(t+9)$

23. $3 \cdot \frac{28}{a} + t^2$

24. $4t - xt$

25. $x(19 - 7a)$

26. $\frac{b+10}{x} - (4 \cdot 3t)$

27. $12 - x^2$

28. $x - 12$

29. $2b - 3a + 5t$

Preparation:

30. If the formula for conversion between Celsius and Fahrenheit is

$$C = \tfrac{5}{9}(F-32)$$

What is the temperature in Celsius if it is 59° F?

31. If the formula for the amount of money you have in nickels and quarters is

$$M = .05n + .25q$$

How much money is totaled in 15 nickels and 7 quarters?

Answers:

1. $-18\frac{1}{8}$

2. $14\frac{59}{80}$

3. $127\frac{31}{40}$

4. 55,780.4878

5. 12.745

6. 5.0879

7. $0.107

8. 20.5 miles per hour

9. $47.36 and $477.86

10. $332.85 and $618.15

11. 91.3%

12. 24

13. 40t

14. -58

15. 230

16. 54

17. 8m + 26 or 26 + 8m

18. 7w + 48 or 48 + 7w

19. m + 33 or 33 + m

20. r + 90 or 90 + r

21. 8

22. 26

23. 28

24. -4

25. -150

26. -43

27. -13

28. -7

29. 29

30. In class

31. In class

Section 3.3

Formulas

The biggest thing to remember about variables is that they represent numbers. Whatever arithmetic we did with numbers can be done with variables as well. If we bring back the formula for the perimeter of a rectangle:

$$P = l + w + l + w$$

Could we write it as this?

$$P = l + l + w + w$$

Sure we could.

Notice how when we used to write 3+3, we could rewrite it as 2x3 or 2(3). Now we have w + w which we could write as 2w. Thus our formula for the perimeter of a rectangle becomes:

$$P = \underbrace{l + l} + \underbrace{w + w}$$
$$P = 2l + 2w$$

Through the same process we can come up with many formulas to use. Though it has all been made up before, there is much to gain from knowing where a formula comes from and how to make them up on your own. I will show you on a couple of them.

Distance, rate

If you were traveling at 40mph for 2 hours, how far would you have traveled? Well, most of you would be able to say 80 mi. How did you come up with that? Multiplication:

$$(40)(2) = 80$$

(rate of speed) · (time) = distance

or in other words:

$$rt = d$$

where r is the rate
t is the time
d is the distance

Percentage

If you bought something for $5.50 and there was a 8% sales tax, you would need to find 8% of $5.50 to find out how much tax you were being charged.

$$.44 = .08(5.50)$$

Amount of Tax = (interest rate) · (Purchase amount)

or in other words:

$$T = rP$$

Where t is tax
r is rate of tax
P is the purchase amount.

Simple Interest

This formula is very similar to another one that involves simple interest. If you invested a principal amount of \$500 at 9% interest for three years, the amount of interest would be given by the formula:

$$I = Prt$$

where I is the interest earned
P is the principal amount (starting amount)
r is the interest rate
t is the time that it is invested.

Temperature Conversion

Most of us know that there is a difference between Celsius and Fahrenheit degrees, but not everyone knows how to get from one to the other. The relationship is given by:

$$C = \frac{5}{9}(F - 32)$$

where F is the degrees in Fahrenheit
C is the degree in Celsius

Money

If you have a pile of quarters and dimes, each quarter is worth 25¢ (or \$.25) and each dime is worth 10¢ (\$.10), then the value of the pile of coins would be:

$$V = .25q + .10d$$

where V is the Total Value of money
q is the number of quarters
d is the number of dimes

Examples

Section 3.3 Exercises

Add or Subtract. Simplify.

2.2

1. $6\frac{7}{8} - 13\frac{3}{8} =$

2. $7\frac{5}{12} + 187\frac{3}{4} =$

3. $21\frac{5}{6} - 97\frac{2}{15} =$

Divide.

2.4

4. $3.7\overline{)9.574}$

5. $6000\overline{)254.7}$

6. $.008\overline{)37.65}$

2.5

7. If a wood floor costs $4.50 per square foot, how much is that per square yard?

8. How much does it cost to run a 700 watt microwave for 17 hours per week if the power company charges 12 cents per kilowatt-hour?

Find the following:

2.6

9. Price: $39.48
Tax rate: 5%
Tax:
Total Price:

10. Price: $2,736.00
Percent off: 35%
Amount saved:
Final Price:

11. Birds: 140
Black : 47
Percent of black birds:

Find the following:

3.1

12. $4 \cdot 3 - 8(9)$

13. $5 \cdot d \cdot 7 \cdot p$

14. $5(3-9) - 2^3 \cdot (5+4)$

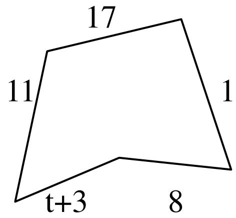
15. $45(7.8)$

16. $3 \cdot 7 \cdot m \cdot 2$

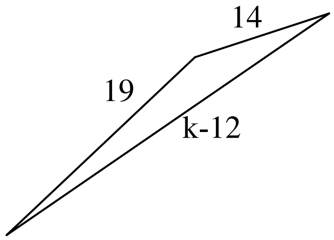
17 $2(3^2)+5(4)+8 \cdot m$

Find the perimeter of the following shapes:

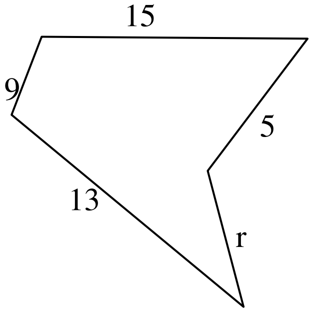
18.



19.



20.



Find the following when m = 3, n = 7, t = 15, and a = 4.

3.2

21. 3t - 7

22. 2(n+9)

23. $3 \cdot \frac{28}{a} + m^2$

24. $12 - a^3$

25. m – 12

26. 2n – 3a + 5t

Use the formula for distance, rate and time.

3.3

Example:
$r = 3$
$t = 14$
$d =$

→

Formula is found on page 153: **$rt = d$**

$3(14) = d$
$42 = d$

27. $r = 7$
$t = 15$
$d =$

28. $r = 55$
$t = 7.2$
$d =$

29. $r = 45$
$t = 2\frac{1}{3}$
$d =$

Use the formula for Simple Interest.

30. $P = 5{,}000$
$r = 7\%$
$t = 5$
$I =$

31. $P = 8{,}500$
$r = 4\%$
$t = 9$
$I =$

32. $P = 300$
$r = 9\%$
$t = 15$
$I =$

Use the formulas for Money totals (you may have to make up your own) when q stands for quarters, d for dimes, n for nickels and p for pennies.

33. $q = 9$
$d = 12$
$V =$

34. $p = 19$
$d = 17$
$V =$

35. $n = 37$
$q = 23$
$V =$

Use the formulas for Temperature Conversion.

36. $F = 75$
$C =$

37. $F = 15$
$C =$

38. $F = -23$
$C =$

Preparation:

39. If the formula for area of a circle is

$$A=\pi r^2$$

What is the area of a circle with radius 7?

Answers:

1. $-6\frac{1}{2}$

2. $195\frac{1}{6}$

3. $-75\frac{3}{10}$

4. 2.5876

5. .04245

6. 4,706.25

7. $40.50

8. $1.43

9. $1.97 and $41.45

10. $957.60 and $1,778.40

11. 33.6%

12. -60

13. 35dp

14. -102

15. 351

16. 42m

17. 38 + 8m or 8m + 38

18. 58 + t or t + 58

19. k + 21 or 21 + k

20. 42 + r or r + 42

21. 38

22. 32

23. 30

24. -52

25. -9

26. 77

27. 105

28. 396

29. 105

30. $1,750

31. $3,060

32. $405

33. $3.45

34. $1.89

35. $7.60

36. 48.2° C

37. -9.4° C

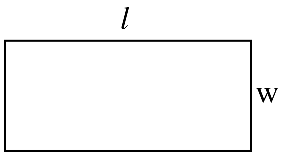
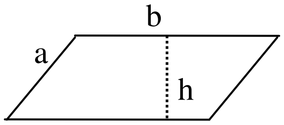
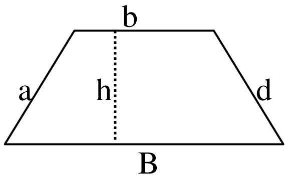
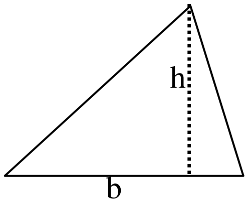
38. -30.6° C

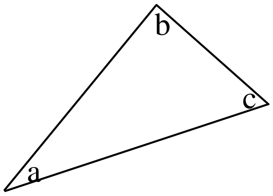
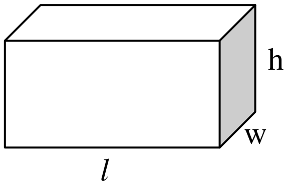
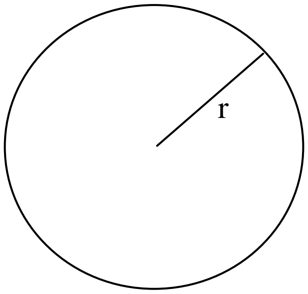
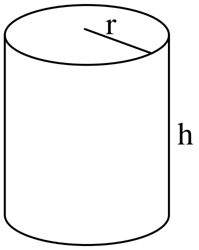
39. In class

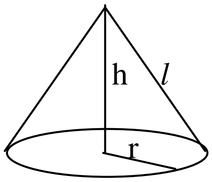
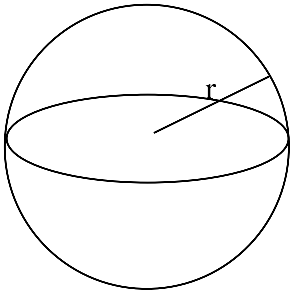
Section 3.4
Geometry Formulas

There are tons of formulas out there. Once you know what numbers you are going to multiply or divide, you can make your own. Luckily, we don't have to make all of them up and some nice people in the past have gone to much work to do that for us. I have provided some formulas that describe certain things about shapes.

Shape formulas:

 Rectangle	$P = 2l + 2w$ $A = lw$	**P** is the perimeter ***l*** is the length **w** is the width **A** is the Area
 Parallelogram	$P = 2a + 2b$ $A = bh$	**P** is the perimeter **a** is a side length **b** is the other side length **A** is the Area
 Trapezoid	$P = b+a+B+d$ $A = \frac{1}{2}h(B+b)$	**P** is perimeter **b** is the little base **B** is the big base **a** is a leg **d** is a leg **A** is the Area
 Triangle	$P = s_1+s_2+s_3$ $A = \frac{1}{2}bh$	**P** is the perimeter **A** is the Area

 Triangle	$a + b + c = 180$	**a** is one angle **b** is another angle **c** is another angle
 Rectangular Solid	$SA = 2lw + 2wh + 2lh$ $V = lwh$	***l*** is the length **h** is the height **w** is the width **SA** is the Surface Area **V** is volume
 Circle	$C = 2\pi r$ $A = \pi r^2$	**C** is the Circumference or Perimeter **π** is a number, about 3.14159… it has a button on your calculator **r** is the radius of the circle **A** is the area inside the circle.
 Cylinder	$LSA = 2\pi rh$ $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	**LSA** is Lateral Surface Area = Area just on the sides **h** is the height **SA** is total surface area **π** is a number, about 3.14159… it has a button on your calculator **r** is the radius of the circle **V** is Volume

 Cone	$\text{LSA} = \pi r l$ $\text{SA} = \pi r^2 + \pi r l$ $V = \frac{1}{3}\pi r^2 h$	**h** is the height **r** is the radius of the circle ***l*** is the slant height **π** is a number, about 3.14159… it has a button on your calculator **SA** is total surface area **LSA** is Lateral Surface Area = Area just on the sides **V** is the Volume
 Sphere	$\text{SA} = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	**r** is the radius **SA** is the surface area **V** is the Volume

Section 3.4 Exercises

Find the following:

3.1

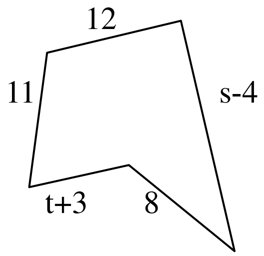
1. $7 + 6 \cdot 3 - 8(5)$

2. $2 \cdot v \cdot 9 \cdot m$

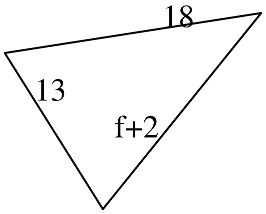
3. $6(3-7) - 4^2 \cdot (7+4)$

Find the perimeter of the following shapes:

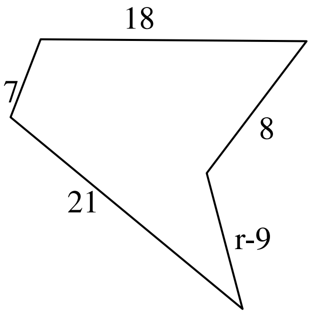
4.



5.



6.



Find the following when p = -5, r = 7, t = $\frac{2}{3}$, and a = 4.

3.2

7. $12 - a^3$

8. $\frac{7a - 12}{4}$

9. $2r - 3p + 9t$

Use the formula for distance, rate and time.

3.3

10. r = 6
t = 19
d =

11. r = 65
t = 4.3
d =

12. r = 36
t = $2\frac{1}{3}$
d =

Use the formula for Simple Interest.

13. P = 2,800
r = 7%
t = 4
I =

14. P = 136,000
r = 6%
t = 9
I =

15. P = 300
r = 13%
t = 7
I =

Use the formulas for Money totals (you may have to make up your own) when q stands for quarters, d for dimes, n for nickels and p for pennies.

16. q = 15
d = 27
V =

17. p = 30
d = 25
V =

18. p = 37
q = 23
n = 7
V =

Use the formula for Temperature Conversion.

19. F = 212
C =

20. F = 98.6
C =

21. F = -40
C =

Use the formulas for a cone (use a calculator).

3.4

Example:
$r = 3$
$l = 14$
SA =

Formula is found on page 162: $SA = \pi r^2 + \pi r l$
$\pi(3^2) + \pi(3)14$
$9\pi + 42\pi$
$28.274 + 131.947$
$\boxed{160.22}$

22. $r = 6$
$h = 7$
V =

23. $r = 9$
$l = 12.8$
SA =

24. $r = 3$
$l = 7.9$
LSA =

Use the formulas for a rectangle (no calculator).

25. $l = 3$
$w = 5$
A =

26. $l = 10.7$
$w = 4$
A =

27. $l = 8.6$
$w = 9$
P =

Use the formulas for a circle (use a calculator).

28. $r = 4$
C =

29. $r = 15$
A =

30. $r = 7$
C =

Use the formulas for rectangular solid.

31. $l = 7$
$w = 2$
$h = 8$
SA =

32. $l = 4.2$
$w = 5$
$h = 7$
V =

33. $l = 6$
$w = 8$
$h = 2\frac{1}{2}$
SA =

Preparation:

34. Combine.

5 penguins + 6 penguins =

$5p + 6p =$

35. Combine everything that is alike:

$3x + 2a - 7x + 10a - 3b$

Answers:

1. -15
2. $18vm$
3. -200
4. $30 + s + t$
5. $f + 33$
6. $r + 45$
7. -52
8. 4
9. 35
10. 114
11. 279.5
12. 84
13. $784
14. $73,440
15. $273
16. $6.45
17. $2.80
18. $6.47
19. 150° C
20. 37° C
21. -40° C
22. 263.89
23. 616.38
24. 74.46
25. 15
26. 42.8
27. 35.2
28. 25.13
29. 706.86
30. 43.98
31. 172
32. 147
33. 166
34. In class
35. In class

Section 3.5
Combining

Now remember that formulas are simply generalizations of what is done repeatedly. If there is something that you do over and over, you can make up your own formula. Here, I will show you how.

Most of you go shopping. Where I live, the sales tax is currently set at 6%. We have already seen that if I want to get how much tax I will pay when I purchase something, I will have:

$$T = rP$$

In this case the tax will always equal

$.06P$

But now, how do I find out the final price of something with the tax added on? Let's look.

Price: \$15.00
Tax: (.06)(\$15.00) = \$.90

For the total price we would add them together.

Total price: \$15.90

Again. What if I were to buy something for \$56.00?

Price: \$56.00
Tax: (.06)(\$56.00) = \$3.36

For the total price we would add them together.

Total price: \$59.36

In general, we always add the tax to the original price to get the final price.

$$T = P + .06P$$

That looks slick, but there is even more that we can do. Do you remember way back when you were learning to add things together, the biggest rule was that you have to **add like things.** For example,

Notice that when we have just an x or a rock, it means 1x or 1rock.

4penguins + 3penguins = 7penguins
5rocks + another rock = 6 rocks
$7t + 8t = 15t.$
$3.4m + 9m = 12.4m$
$1.35x + x = 2.35x$

Whenever you have **like things you can add them**. Thus our formula can be even shorter:

$$T = P + .06P$$

$$T = 1.06P$$ We just combine the P's

This is called **combining like terms**. Whenever you have a bunch (or couple) of numbers or letters multiplied together, it is called a term. If all but the numbers match up exactly to make them alike, then you can combine them.

Example:

$5x - 13x$
$-8x$

Example:

$2qy + 59.4qy$
$61.4qy$

Example:

$3x - 19x$

$-22x$

There are some rules that are obvious when working with numbers, but they aren't so obvious now that we are working with letters. We have always been able to switch numbers places while adding them or multiplying them:

Addition	**Multiplication**
$5+3 = 3+5$	$3(5)=5(3)$

Well, the same idea holds with variables:

$x+7 = 7+x$	$x7 = 7x$

Since this is always true with addition and multiplication, we give it a name:

$3xy + 7t = 7t + 3xy$ **Commutative** $4mkl = 4kml$

Another one that has helped us is that we can add or multiply things in any order that we need to:

(3+5) + 7 where we add the 3 and 5 together first and then add the 7 will always be the same as 3 + (5+7) where we add the 5 to the 7 first and then add on the 3. Changing which number the 5 associates with first, without changing the answer is called:

$x + (7+t) = (x+7) + t$ **Associative** $3(x5) = (3x)5$

Addition and multiplication both had numbers that didn't do anything. They are called:

$6 + 0 = 6$ **Identity** $7(1) = 7$

And each number has it's opposite (or inverse) when dealing with addition and multiplication:

$7 + (-7) = 0$ **Inverse** $7 \cdot \frac{1}{7} = 1$

And lastly, there was one law that put Addition and Multiplication together. We used it to multiply large numbers together. It was called:

Distributive

$5(43) = 5(40) + 5(3)$

$7(x - 4) = 7x - 28$

All of these will help you be able to simplify and combine terms with variables in them.

Name	Operation	What it does	Example
Commutative	Addition	We can swap numbers around addition	5 + 7 7 + 5 are the same. 2x + 3y 3y + 2x are the same.
Commutative	Multiplication	We can swap numbers around multiplication	7xtzy x7tzy are the same
Associative	Addition	We can begin addition with any of the entries.	7+(5+1) (7+5)+1 are the same (x+5)+9 x+(14) are the same
Associative	Multiplication	We can begin multiplication with any of the entries	$7\cdot(3\cdot 2)$ $(7\cdot 3)\cdot 2$ are the same
Associative and Commutative together	Either Multiplication or Addition	We can multiply (or add) in any order we want to.	
Identity	Addition	0 is invisible in addition	6+0=6 x+0=x
Identity	Multiplication	1 is invisible in multiplication	$6\cdot 1=6$ $x\cdot 1=x$
Inverse	Addition	Opposite when adding.	6 -6 3t -3t are opposites -17 17 add to zero.
Inverse	Multiplication	Opposite when multiplying (inverses)	-5 $-\frac{1}{5}$ $\frac{2}{3}$ $\frac{3}{2}$ multiply to 1. 17 $\frac{1}{17}$
Distributive	Both	Jump numbers into parentheses	6(43) = 6(40+3) = 6(40) + 6(3) 7(2x – 5) = 14x – 35

These are the same rules that you have been using then entire time you have added, subtracted multiplied and divided numbers. All we have done now is given them names.

Examples

Section 3.5 Exercises

Find the following when p = 5, r = -7, t = $\frac{2}{3}$, and a = 3.

3.2

1. $12 - a^3$

2. $\frac{10a - 12}{3 \cdot 6}$

3. $2r - 3p + 15t$

Use the formula for Simple Interest.

3.3

4.
P = 5,000
r = 6.2%
t = 4
I =

5.
P = 45,000
r = 6%
t = 9.3
I =

6.
P = 1,300
r = 8.9%
t = 7
I =

Use the formulas for Money totals (you may have to make up your own) when q stands for quarters, d for dimes, n for nickels and p for pennies.

7.
q = 15
d = 27
n = 13
V =

8.
p = p
d = d
V =

9.
p = p
q = q
n = n
V =

Use the formula for Temperature Conversion.

10.
F = 3,000
C =

11.
F = 45
C =

12.
F = -273
C =

Use the formulas for a sphere (use a calculator).

3.4

13.
r = 6
V =

14.
r = 9
SA =

15.
r = 3
SA =

Use the formulas for a triangle (no calculator).

16.
b = 3
h = 5
A =

17.
b = 10.7
h = 4
A =

18. Two angles are 47° and 69°; what is the third?

Use the formulas for a trapezoid (no calculator).

19.
b = 6
B = 10
h = 7
A =

20.
b = 3
B = 15
h = 3
A=

21.
b = 7
B = 10
a = 12
d = 8
P =

Use the formulas for a parallelogram (no calculator).

22. b = 6
h = 7
A =

23. b = 3
h = 3
A=

24. b = 7
a = 12
P =

Simplify.

3.5

Example:
$2(x-7)-3(x-8)$

$2(x-7)-3(x-8)$
$2x-14-3x+24$
$-x+10$

25. $7y+5y$

26. $7a-2+4a$

27. $18r-7t+8t+6r$

28. $2(x-5)+7x$

29. $8t+4(t+12)$

30. $8-3(7-9t)+4t$

31. $8-3x^2+5+10x^2$

32. $8x^2-5x-4x^2+13x$

33. $2xy+7x(5y-4)$

Identify the property that is illustrated by each statement.

34. $3+x=x+3$

35. $6\cdot\frac{1}{6}=1$

36. $18x(x-3)=18x^2-54x$

37. $2x(5x\cdot\frac{1}{n})=(2x5x)\frac{1}{n}$

38. $8x+(7t+9x)=8x+(9x+7t)$

39. $18z+0=18z$

40. $25m+(-25m)=0$

41. $2t(27+x)=(27+x)2t$

Answers:

1. -15

2. 1

3. -19

4. $1240

5. $25,110

6. $809.90

7. $7.10

8. .01p +.10d

9. .01p+.25q+.05n

10. 1,648.9° C

11. 7.2° C

12. -169.4° C

13. 904.78

14. 1017.88

15. 113.1

16. 7.5

17. 21.4

18. 64°

19. 56

20. 27

21. 37

22. 42

23. 9

24. 56

25. 12y

26. 11a – 2

27. 24r + t

28. 9x – 10

29. 12t + 48

30. 31t – 13

31. $7x^2 +13$

32. $4x^2 + 8x$

33. 37xy – 28x

34. Commutative of Addition

35. Multiplicative Inverse

36. Distributive

37. Associative of Multiplication

38. Commutative of Addition

39. Additive Identity

40. Additive Inverse

41. Commutative of Multiplication

Chapter 3 Review Exercises (1)

Find the following when p = 8, r = -7, t = $\frac{2}{3}$, and a = 3.

3.2

1. $12 + a^3$

2. $\frac{10a - 12}{3r}$

3. $5r - 7p + 6t$

Use the formula for Simple Interest.

3.3

4. P = 520
r = 6.2%
t = 4
I =

5. P = 35,000
r = 6%
t = 9.3
I =

6. P = 19,000
r = 8.9%
t = 7
I =

Use the formulas for Money totals (you may have to make up your own) when q stands for quarters, d for dimes, n for nickels and p for pennies.

7. q = 25
d = 17
n = 15
V =

8. p = p
d = q-13
V =

9. p = p
q = q
n = q+7
V =

Use the formula for Temperature Conversion.

10. F = 300
C =

11. F = -45
C =

12. F = 102
C =

Use the formulas for a cone (use a calculator).

3.4

13. r = 6
h = 11
V =

14. r = 9
l = 5
SA =

15. r = 3
l = 8
LSA =

Use the formulas for a triangle (no calculator).

16. b = 24
h = 5
A =

17. b = 15
h = 4
A =

18. Two angles are 37° and 81°; what is the third?

Use the formulas for a trapezoid (no calculator).

19. b = 7
B = 10
h = 7
A =

20. b = 9
B = 15
h = 3
A=

21. b = 7
B = 15
a = 12
d = 8
P =

Use the formulas for a rectangular solid (no calculator).

22. $l = 6$
$w = 9$
$h = 7$
SA =

23. $l = 4$
$w = 15$
$h = 7$
SA =

24. $l = 6$
$w = 14$
$h = 2$
V =

Simplify.

3.5

25. $8y + 5y$

26. $4a - 9 + 4a$

27. $16r - 5t + 3t + 12r$

28. $7(x - 5) + 15x$

29. $7t + 4(t + 12)$

30. $8 - 6(7 - 4t) + 4t$

31. $8 - 12x^2 + 5 + 3x^2$

32. $7x^2 - 5x - 9x^2 + 13x$

33. $13xy + 7x(6y - 4)$

Identify the property that is illustrated by each statement.

34. $(3 + x) + 9 = 3 + (x + 9)$

35. $18x(x - 3) = 18x^2 - 54x$

36. $8x + (7t + 9x) = 8x + (9x + 7t)$

37. $18z + (-18z) = 0$

38. $5m(27 + x) = (27 + x)5m$

39. $6 \cdot 1 = 6$

40. $25m + 0 = 25m$

41. $2x(5x \cdot \frac{1}{n}) = (2x5x)\frac{1}{n}$

Answers:

1. 39

2. $-\frac{6}{7}$

3. -87

4. $128.96

5. $19,530

6. $11,837

7. $8.70

8. $V = .01p + .1(q-13)$

9. $V = .01p + .3q + .35$

10. 148.9°

11. -42.8°

12. 38.9°

13. 132π or 414.69

14. 126π or 395.84

15. 24π or 75.4

16. 60

17. 30

18. 62°

19. 59.5

20. 36

21. 42

22. 318

23. 386

24. 168

25. $13y$

26. $8a – 9$

27. $28r – 2t$

28. $22x – 35$

29. $11t + 48$

30. $28t – 34$

31. $-9x^2 + 13$

32. $-2x^2 + 8x$

33. $26xy – 28x$

34. Associative Addition

35. Distributive

36. Commutative Addition

37. Additive Inverse

38. Commutative Multiplication

39. Multiplicative Identity

40. Additive Identity

41. Associative Multiplication

Chapter 3 Review Exercises (2)

Find the following when $f = 5, r = -7, t = \frac{2}{3}$, and $a = -2$.

3.2

1. $6t - f^3$

2. $\frac{10a - 12}{2f} + t$

3. $2fr - 31a + 15a$

Use the formula for Simple Interest.

3.3

4.
P = 15,000
r = 6.2%
t = 7
I =

5.
P = 23,000
r = 6%
t = 8.7
I =

6.
P = 1,300
r = 8.9%
t = 7
I =

Use the formulas for Money totals (you may have to make up your own) when q stands for quarters, d for dimes, n for nickels and p for pennies.

7.
q = t+5
d = m
n = 13
V =

8.
p = 15
d = 9
V =

9.
p = h+9
q = 7
n = x - 20
V =

Use the formula for Temperature Conversion.

10.
F = -20
C =

11.
F = 59
C =

12.
F = 32
C =

Use the formulas for a cylinder (use a calculator).

3.4

13.
r = 6
h = 12
V =

14.
r = 9
h = 5
SA =

15.
r = 3
h = 8
LSA =

Use the formulas for a triangle (no calculator).

16.
b = 6
h = 5
A =

17.
b = 15
h = 4
A =

18. Two angles are 45° and 79°; what is the third?

Use the formulas for a trapezoid (no calculator).

19.
b = 9
B = 11
h = 7
A =

20.
b = 8
B = 15
h = 5
A=

21.
b = 12
B = 25
a = 13
d = 17
P =

Simplify.

3.5

22. $9y - 11y$

23. $10a - 2b + 4a - 9b$

24. $8(r - 7t) + 8(t + 6r)$

25. $2(x - 5) + 7$

26. $8m + 4(m + 15t)$

27. $9 - 5(6 - 9p) + 4p$

28. $8x^2 - 34x^3 + 9x^2 + 10x^3$

29. $12x^4 - 5x - 4x^4 + 13x$

30. $3xy - 7x(5y - 4m)$

Identify the property that is illustrated by each statement.

31. $-7\left(-\frac{1}{7}\right) = 1$

32. $2t(13 + f) = (13 + f)2t$

33. $3m + (-3m) = 0$

34. $7x(9x \cdot \frac{1}{n}) = (7x9x)\frac{1}{n}$

35. $\frac{18pz}{\pi\sqrt{17}} \cdot 1 = \frac{18pz}{\pi\sqrt{17}}$

36. $3z + 0 = 3z$

37. $5q + (7t + 9q) = 5q + (9q + 7t)$

38. $9f(x - 3) = 9fx - 27f$

39. Create a visual chart of the rules and procedures regarding variables, their notation, and how to combine them.

Preparation:

40. Find what p is in the following:

$3p = 21$ $\quad$ $p + 7 = 12$ $\quad$ $p - 19 = 39$

Answers:

1. -121

2. $-\frac{38}{15}$ or $-2\frac{8}{15}$

3. -38

4. $6,510

5. $12,006

6. $809.90

7. .25t + .1m + 1.9

8. $1.05

9. .01h + .05x + .84

10. -28.9° C

11. 15° C

12. 0° C

13. 1,357.17

14. 791.68

15. 150.8

16. 15

17. 30

18. 56°

19. 70

20. 57.5 or $\frac{115}{2}$

21. 67

22. -3y

23. 14a – 11b

24. 56r – 48t

25. 2x – 3

26. 12m + 60t

27. 49p – 21

28. $-24x^3+17x^2$

29. $8x^4 + 8x$

30. -32xy + 28xm

31. Multiplicative Inverse

32. Commutative of Multiplication

33. Additive Inverse

34. Associative of Multiplication

35. Multiplicative Identity

36. Additive Identity

37. Commutative of Addition

38. Distributive

39. Turned in

40. In class.

Chapter 4:
LINEAR EQUATIONS

Overview

Section 4.1

Linear Equations

3+ “what” = 7? If you have come through arithmetic, the answer is fairly obvious: 4.

However, if I were to ask something like:
2 times “what” plus 5 all divided by 7, then minus 6 = 5? There tends to be a little more difficulty in popping out the answer. The beauty of math is that it allows us to write down all of that stuff and then systematically make it simpler and simpler until we have only the number left. Wonderful.

We start with the easy ones to find out all of the rules and then we will build up to the big ones.

3+ “what” = 7

First, we need to adjust the fact that we are going to be writing “what” all the time. A very common thing is to put a letter in that place that could represent any number. We call that a **variable**. We replace the word “what” with “x” (or you could use p, q, r, f, m, l . . .) So our equation becomes:

$$3 + x = 7$$

The whole goal of Algebra is to find the number that makes that statement true. We already know that the number is 4. We would write:

$$x = 4$$

Now, look at what happened to our original equation. Do you see that the right side is missing a 3 and the left side is now 3 lower as well. This gives us some insight into what we can do to equations! Try another one:

$$x + 8 = 10$$

What number would make that statement true? If x were equal to 2, it would work. We write:

$$x = 2$$

Notice how we get the number that would work by subtracting that 8 from both sides of the equation. Let’s see if it works with some other equations:

$x – 7 = 2$ $\qquad$ $x – 3 = 10$

With these two equations, the answers are:

$x = 9$ $\qquad$ and $\qquad$ $x = 13$

We got the answers by adding the 7 and the 3 to the right hand sides. This brings up a good point. In the first couple of equations that we did, we subtracted when the equation was adding. In the next two equations, we added when the equation was using subtraction. Let’s look at what happens when we start doing multiplication:

$$4x = 20.$$

What number would work? That is right, 5.

$$x = 5$$

What would you do to 20 to get 5? Divide by 4. Holy smokes! That is the exact opposite of what the equation is doing. Here is another:

Examples

$$\frac{x}{7} = 4$$

What number divided by 7 equals 4? That's it, 28. We times 4 by 7 to get that answer. Multiplying by 7 is the exact opposite of dividing by 7.

This leads us to a couple of conclusions that form the basis for everything we will do in Algebra:

1) **When we want to get rid of numbers that are surrounding the variable, we need to do the opposite (technically called the inverse) of them.**

2) **We can add, subtract, multiply, or divide both sides of an equation by any number and still have the equation work.**

A great way to think about these concepts is as though you have a balance that is centered on the equal sign. As long as you put the same thing on both sides, you remain balanced.

Here is how it would work, one of each:

$$\begin{array}{llll} x+7=11 & 4x=24 & x-3=24 & \frac{x}{5}=7 \\ \;\;-7\;\;\;-7 & \;/4\;\;\;/4 & \;\;+3\;\;\;+3 & (5)\;\;(5) \\ & & & \\ x = 4 & x = 6 & x = 27 & x = 35 \end{array}$$

You may ask why we go through all of that when the answers are obvious. The answer is that these problems will not be so easy later on, and we need to practice these easy ones so that when we get the hard ones, they crumble before our abilities. Now to some which are a little tougher.

When we have one like this:

$$2x - 7 = 11$$

We could think about it long enough to find a number that works, and maybe you can do that, but I have to tell you that in just a little while we are going to have a problem that you won't be able to do that with too quickly. So, let's use what we learned to get rid of the 2 and the 7 so that x will be left by itself. If you remember the order of operations, you will remember that the 2 and the x are stuck together by multiplication, so we can't get rid of the 2 until the 7 has been taken care of like this:

$2x - 7 = 11$

$2x = 18$ (we added 7 to both sides)

$x = 9$ (divided both sides by 2)

To illustrate the idea of un-doing operations, I would like to try to stump you with math tricks.

We begin. I am thinking of a number, and it is your job to guess what the number is.

I am thinking of a number.
I times the number by two.
I get 10.

Not too hard to figure out, you say? You're right. The answer is 5 and you obtained that by taking the result and going backwards. Try the next one:

I am thinking of a number.
I times the number by 3.
Then I subtract 5.
Then I divide that number by 2.
Then I add 4 to that.
I get 18.
What was the number I started with?

Aha. A little tougher don't you think? Well, If you think about it just one step at a time, then the thing falls apart. What number would I add 4 to to get 18? 14 (notice that it is just 18 **subtract** 4). We can just follow **up** the line doing the exact opposite of what I did to my number. Here you go:

Start with 18
Subtract 4 = 14
Multiply by 2 = 28
Add 5 = 33
Divide by 3 = 11.

That's it! Most of Algebra is summed up in the concept of un-doing what was done.

I am thinking of a number.
I times it by 4.
Then I add 5.
Then I divide by 9.
Then I subtract 7.
I get -2. What did I start with?

This one is done the same way as the other one but I wanted to show you how you make that into an equation that will be useful in the rest of your math career. Instead of writing each step out, we construct an equation. We write it again but this time we will write the equation along with it:

I am thinking of a number.	*We call that x.*
I times it by 4.	$4x$
Then I add 5.	$4x+5$
Then I divide by 9.	$\frac{4x+5}{9}$
Then I subtract 7.	$\frac{4x+5}{9}-7$
I get -2. What did I start with?	$\frac{4x+5}{9}-7=-2$

That looks like a nasty equation, but it is done in exactly the same way. We just go backwards and un-do all of the things that were done to the original number. **We are using the rule that we can add, subtract, multiply or divide both sides of the equation by the same thing.**

I know you can do it when it is all written out, so I will show you what it looks like using the equation:

$\frac{4x+5}{9}-7=-2$	
$\frac{4x+5}{9}=5$	add 7 to both sides
$4x+5=45$	times both sides by 9
$4x=40$	subtract 5 from both sides
$x=10$	divide both sides by 4.

Notice here that we are still undoing in the opposite order of what was there.

10 is the number I started with! Go ahead and make sure by sticking it into the original problem, and you will see that we found the right number. We call that number a **solution**, because it is the only number that **solves** the equation.

Section 4.1 Exercises

Find the Volume of a rectangular solid when the width, height and length are given.
Formula is V=*l*wh

3.4

1. $l = 4$ in
w = 2.5 in
h = 3 in
V =

2. $l = 7$ ft
w = 4 ft
h = 2.8 ft
V =

3. $l = 7.2$ m
w = 9 m
h = 3 m
V =

Find the Area of a trapezoid when the bases and height are given. Formula is
$A = \frac{1}{2}h(B+b)$

4. B = 15
b = 10
h = 7
A=

5. B = 21
b = 11
h = 3
A=

6. B = 19
b = 6
h = 10
A=

Identify the property that is illustrated by each statement:

3.5

7. $(8+5)+3 = 3+(8+5)$

8. $(3xy)7x = (3yx)7x$

9. $-18 + 0 = -18$

10. $(8ab)7c = 8(ab7)c$

11. $6 \cdot \frac{1}{6} = 1$

12. $(x+5)+7 = x+(5+7)$

Simplify.

13. $2(3+x)+5(x-7)$

14. $5(a-3b) - 4(a-5)$

15. $3x+4y-7z+7y-3x+18z$

16. $2s(t-7) - 6t(s+3)$

17. $3(x^2-5n) + 3n - 7x^2$

18. $6kj - 7k + 8kj + 11$

Solve.

4.1

Example:

$4x + x - 7 = 1$	Combine x's
$5x - 7 = 1$	+7 on both sides
$5x = 8$	
$x = \frac{8}{5}$	Divide by 5 on both sides

19. $5\left(\frac{3x-1}{7}+2\right) = 35$

20. $r + 9 = -15$

21. $-3 + m = 18$

22. $\frac{7}{3}t = 14$

23. $-13 = 5x + 7$

24. $\frac{5x-6}{4} = 3$

25. $-\frac{3}{8}x - 4 = 20$

26. $12 + 2p = 3$

27. $.4y = 78$

28. $5x + 3 - 7x = 15$

29. $3x - 9 + 2x = -3$

30. $.3p + 5 = 19$

31. $3\left(\frac{-2x-8}{6} + 7\right) - 3 = 12$

32. $4f + 9 = 9$

33. $\frac{2x+3}{5} = 11$

34. $t + t + 4t - 7 = 17$

35. $3\left(\frac{5x-8}{6} + 7\right) - 3 = 18$

Preparation.

36. After reading some from Section 4.2, Try to solve this problem.
68 is 13 more than 5 times a number. What is the number?

37. Find the missing variable for a rectangle:
w = 5
l =
P = 35.4

Answers:

1. 30 in^3
2. 78.4 ft^3
3. 194.4 m^3
4. 87.5
5. 48
6. 125
7. Commutative property of addition
8. Commutative property of multiplication
9. Additive identity
10. Associative property of multiplication
11. Multiplicative inverse
12. Associative property of addition
13. 7x – 29
14. a – 15b + 20
15. 11y + 11z
16. -4st – 14s – 18t
17. $-4x^2 - 12n$
18. 14kj – 7k + 11
19. x = 12
20. r = -24
21. m = 21
22. t = 6
23. x = -4
24. $x = \frac{18}{5}$ or 3.6
25. x = -64
26. $p = -\frac{9}{2}$
27. y = 195
28. x = -6
29. $x = \frac{6}{5}$ or 1.2
30. $p = 46.\overline{6}$
31. x = 2
32. f = 0
33. x = 26
34. t = 4
35. $x = \frac{8}{5}$ or 1.6
36. In class
37. In class

Section 4.2

Applications (1)

Applications of linear equations

"When am I ever going to use this?" "Where would this be applicable?" All the way through math, students ask questions like these. Well, to the relief of some and the dismay of others, you have now reached the point where you will be able to do some problems that have been made out of real life situations. Most commonly, these are called, "story problems".

The four main points to remember are:

D- **Data**. Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

V- **Variable**. In all of these story problems, there is something that you don't know, that you would like to. Pick any letter of the alphabet to represent this.

P- **Plan**. Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. This section of the book is divided up so as to explain most of the different kinds of patterns.

E- **Equation**. Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

Once you have mastered the techniques in solving linear equations, then the fun begins. Linear equations are found throughout mathematics and the real world. Here is a small outline of some applications of linear equations. You will be able to solve any of these problems by the same methods that you have just mastered.

Section 4.2a

Translation

Translation

The first application is when you simply translate from English into math. For example:

Seven less than 3 times what number is 39?

Since we don't know what the number is, we pick a letter to represent it (you can pick what you would like to); I will pick the letter x:

$$3x - 7 = 39 \text{ then solve}$$
$$3x = 46$$
$$x = \tfrac{46}{3} \qquad (\text{or } 15\tfrac{1}{3} \text{ or } 15.3)$$

That's the number.

Section 4.2b
Substitution

Substitution

Sometimes you are given a couple of different things to find. Example:

Two numbers add to 15, and the second is 7 bigger than the first. What are the two numbers?

Pick some letters to represent what you don't know. Pick whatever is best for you. I will choose the letter "f" for the first number and "s" for the second. I then have two equations to work with:

$$f + s = 15 \quad \text{and} \quad s = \boxed{f + 7}$$

The letter "s" and "f+7" are exactly the same and can be changed places.

$$\begin{aligned} f + f + 7 &= 15 \\ 2f + 7 &= 15 \\ 2f &= 8 \\ f &= 4 \end{aligned}$$

4 must be the first number, but we need to stick it back in to one of the original equations to find out what "s" is.

$$\begin{aligned} s &= f + 7 \\ &= 4 + 7 \\ &= 11. \end{aligned}$$

4 and 11 are our two numbers.

These kind of problems often take the form of an object being cut into two pieces. Here, I will show you what I mean.

Example:

A man cuts a 65 inch board so that one piece is four times bigger than the other. What are the lengths of the two pieces?

Now, I would personally pick "f" for first and "s" for second. We know that

$$f + s = 65 \text{ and that } s = 4f$$

Thus,

$$\begin{aligned} f + 4f &= 65 \\ 5f &= 65 \\ f &= 13, \end{aligned}$$

so the other piece must be 52.

The pieces are 13in and 52in.

Example:

A woman has a total of $5.55 in nickels and dimes. The number of dimes is 15 more than the number of nickels. How many dimes and nickels does she have?

I think "d" for dimes and "n" for nickels would work great. So we have that

$$.1d + .05n = 5.55 \text{ and also that } d = n + 15$$

Thus,

$$\begin{aligned} .1(n+15) + .05n &= 5.55 \\ .1n + 1.5 + .05n &= 5.55 \\ .15n &= 4.05 \\ n &= 27 \end{aligned}$$

So we get that there are 27 nickels and then there must be 42 dimes.

Examples

Section 4.2c

Shapes

Shapes

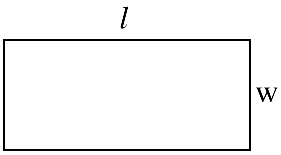
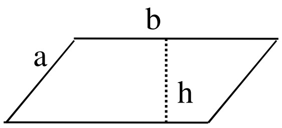
With many of the problems that you will have, pictures and shapes will play a very important role. When you encounter problems that use rectangles, triangles, circles or any other shape, I would suggest a few things:

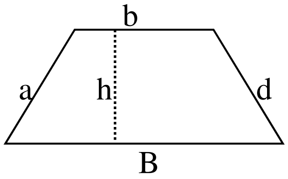
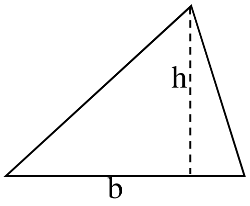
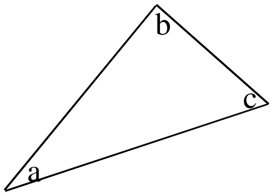
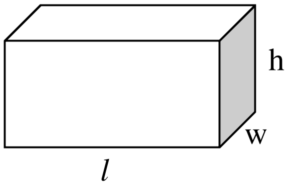
1. Read the problem
2. READ the problem again.
3. READ THE PROBLEM one more time.

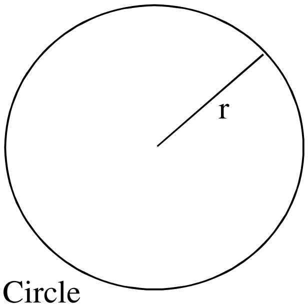
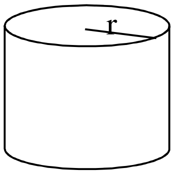
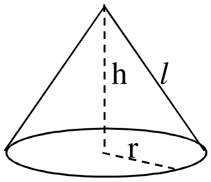
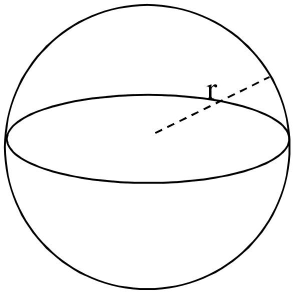
Once you draw a picture to model the problem – read the problem again to make sure that your picture fits.

Here are some formulas for common shapes that you will encounter. You should start to become familiar with them:

Shape formulas:

 Rectangle	$P = 2l + 2w$ $A = lw$	**P** is the perimeter ***l*** is the length **w** is the width **A** is the Area
 Parallelogram	$P = 2a + 2b$ $A = bh$	**P** is the perimeter **a** is a side length **b** is the other side length **h** is height **A** is the Area

 Trapezoid	$P = b+a+B+d$ $A = \frac{1}{2}h(B+b)$	**P** is perimeter **b** is the little base **B** is the big base **a** is a leg **h** is height **d** is a leg **A** is the Area
 Triangle	$P = s_1+s_2+s_3$ $A = \frac{1}{2}bh$	**P** is the perimeter **h** is height **b** is base **A** is the Area
 Triangle	$a + b + c = 180$	**a** is one angle **b** is another angle **c** is another angle
 Rectangular Solid	$SA = 2lw+2wh+2lh$ $V = lwh$	***l*** is the length **h** is the height **w** is the width **SA** is the Surface Area **V** is volume

 Circle	$C = 2\pi r$ $A = \pi r^2$	**C** is the Circumference or Perimeter **π** is a number, about 3.14159 . . . it has a button on your calculator **r** is the radius of the circle **A** is the area inside the circle.
 Cylinder	$LSA = 2\pi rh$ $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	**LSA** is Lateral Surface Area = Area just on the sides **h** is the height **SA** is total surface area **π** is a number, about 3.14159 . . . it has a button on your calculator **r** is the radius of the circle **V** is Volume
 Cone	$LSA = \pi r l$ $SA = \pi r^2 + \pi r l$ $V = \frac{1}{3}\pi r^2 h$	**h** is the height **r** is the radius of the circle ***l*** is the slant height **π** is a number, about 3.14159 . . . it has a button on your calculator **SA** is total surface area **LSA** is Lateral Surface Area = Area just on the sides **V** is Volume
 Sphere	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	**r** is the radius **SA** is the surface area **V** is the Volume

Section 4.2 Exercises

Identify the property that is illustrated by each statement:

3.5

1. $(7+y)+3 = 3+(7+y)$ **2.** $(3xy)7x = (3yx)7x$ **3.** $(5ar)7t = 5(ar7)t$

4. $6 \cdot 1 = 6$ **5.** $-21 + 21 = 0$ **6.** $(p+5)+2 = p+(5+2)$

Simplify.

7. $4s(t-9) - t(s+11)$ **8.** $12(x^2-5n) + 3n - 4x^2$ **9.** $6nj - 7j + 8nj + 11n$

Solve.

4.1

10. $5\left(\frac{6x-4}{5}+2\right)=30$ **11.** $7\left(\frac{-2x+8}{6}+5\right)-2=12$ **12.** $-3 - 7m = 18$

13. $\frac{7}{2}t = -14$ **14.** $-15 = 3x + 9$ **15.** $\frac{2x-7}{3}=33$

16. $t + 5t + 4t - 7 = 17$ **17.** $9\left(\frac{5x-8}{6}+7\right)-3=42$

Solve for the specified variable.

4.2

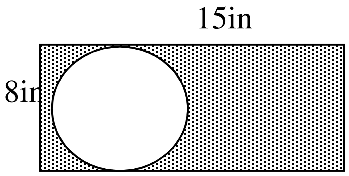
18. 27 is 6 more than 3 times a number. What is the number?

19. 18 less than 5 times a number is 52. What is the number?

20. Two numbers add to 37 and the second is 9 bigger than the first. What are the two numbers?

21. Two numbers add to 238 and the first is 34 bigger than the second. What are the two numbers?

22. Find the area of the shaded region:



23. I have created a triangular garden such that the largest side is 6ft less than twice the smallest and the medium side is 5ft larger than the smallest side. If the total perimeter of the garden is 47ft, what are the lengths of the three sides?

24. If a rectangle's length is 5 more than twice the width and the perimeter is 46 mm, what are the dimensions of the rectangle?

25. A total of $13.45 is made up of dimes and quarters. The number of dimes is 9 less than the number of quarters. How many dimes and quarters are there?

26. If a cone has a Lateral Surface Area of 250 in^2, a radius of 8in, what is the slant height of the cone? Use a calculator.

27. Two numbers add to 588 and the first is 5 times bigger than the second. What are the two numbers?

28. If a cylinder has a volume of 538 cm^3 and a radius of 6 cm, how tall is it? Use a calculator.

29. Find the missing variable for a rectangle:

$P = 39$ ft
$w = 7.2$ ft
$l =$

30. Find the missing variable for a cylinder:

$SA = 800\ \text{in}^2$
$h =$
$r = 9$ in

Preparation.

31. After reading some from Section 4.3, try to solve this equation for x.

x +  = 9 + 

32. Solve the following:

$5x + 9y + 10p= 9y + 15 + 10p$

Answers:

1. Commutative of Addition
2. Commutative of Multiplication
3. Associative of Multiplication
4. Multiplicative Identity
5. Additive Inverse
6. Associative of Addition
7. 3st – 36s – 11t
8. $8x^2 - 57n$
9. 14nj – 7j + 11n
10. x = 4
11. x = 13
12. m = -3
13. t = -4
14. x = -8
15. x = 53
16. $t = \frac{12}{5}$ or 2.4
17. $x = -\frac{4}{5}$
18. 7
19. 14
20. 14, 23
21. 102, 136
22. $69.73\ \text{in}^2$
23. 12, 17, 18
24. l = 17mm; w = 6mm
25. 32 dimes, 41 quarters
26. ℓ = 9.95 in
27. 98, 490
28. 4.76 cm
29. l = 12.3 ft
30. h = 5.15 in
31. In class
32. In class

Section 4.3

More Linear Equations

Unfortunately, not all equations come out so that this un-doing technique works. Sometimes the x shows up in several different places at once:

3x – 5 +2x – 3 = 4x + 7(x – 8)

Seeing all of the x's scattered throughout the equation sometimes looks daunting, but it isn't as bad as all that. We know a couple of ways to make it look a bit more simple.

3x – 5 +2x – 3 = 4x + 7(x – 8) becomes

5x – 8 = 4x + 7x – 56 Distribute the 7 and combine

5x – 8 = 11x – 56 Combine the like terms

Now we reach a point where you should feel somewhat powerful. Remember that you can **add, subtract, multiply or divide** anything you want! (As long as you do it to both sides).

Particularly, I don't like the way that 11x is on the left hand side. I ***choose*** to get rid of it! So, I subtract 11x from both sides of the equation:

5x – 8 = 11x – 56
-11x -11x

Upon combining the like-terms, I get

-6x – 8 = -56

Which now is able to be un-done easily:

-6x = -48 (add 8 to both sides)
x = 8 (divide both sides by -6)

You might as well know that if you didn't like the 5x on the right hand side, you could get rid of that instead:

5x – 8 = 11x – 56
-5x -5x

Combining like terms, we get:
-8 = 6x – 56
48 = 6x (add 56 to both sides)
8 = x (divide both sides by 6)
We will always get the same answer!
You can't mess up!

Special cases: What about 2x + 1 = 2x + 1

Well if we want to get the x's together we had better get rid of the 2x on one side. So we subtract 2x from both sides like this:

2x + 1 = 2x + 1
-2x -2x

1 = 1

0 = 0
5 = 5
-3 = -3
solution is **all real numbers**

Ahh! The x's all vanished.

Well, what do you think about that? This statement is always true no matter what x is. That is the point. x can be any number it wants to be and the statement will be true. **All numbers are solutions**.

0 = 1
5 = 7
-3 = 2
No solution

On the other hand try to solve:
2x + 1 = 2x - 5
-2x   -2x
1 = -5

Again, the x's all vanished. This time it left an equation that is never true. No matter what x we stick in, we will never get 1 to equal -5. It simply will never work. **No solution.**

Solving for a variable:

When given a formula, it is sometimes requested that you solve that formula for a specific variable. That simply means that you are to get that variable by itself.
An example:

Solve for t:
rt = d (Original equation of rate x time = distance)

We are supposed to get t by itself. How do we get rid of the "r"?
Divide both sides by r. It looks like this

rt = d

$$\frac{rt}{r} = \frac{d}{r}$$

$$t = \frac{d}{r}$$ Done. t is by itself.

Another example:

Solve for x:

y = bx +c

y – c = bx subtract "c" from both sides

$$\frac{y-c}{b} = x$$ Divide both sides by "b".

Done. "x" is by itself.

Section 4.3 Exercises

Solve.

4.1

1. $5\left(\frac{3x+4}{5}+2\right)=65$

2. $3\left(\frac{-2x+8}{5}-3\right)+17=20$

3. $-17-7m=-18$

4. $\frac{3}{7}t+1=-11$

5. $9=3x+17$

6. $\frac{5x+7}{4}=13$

7. $8t+3t+14t-17=-17$

8. $7\left(\frac{5x+8}{2}+9\right)-3=18$

4.2

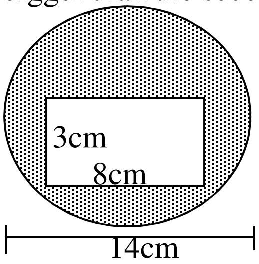
9. 48 is 9 more than 3 times a number. What is the number?

10. 18 less than 7 times a number is 80. What is the number?

11. Two numbers add to 151 and the second is 21 bigger than the first. What are the two numbers?

12. Two numbers add to 436 and the first is 134 bigger than the second. What are the two numbers?

13. Find the area of the shaded region:



14. I have created a triangular garden such that the largest side is 9 less than twice the smallest and the medium side is 7 larger than the smallest side. If the total perimeter of the garden is 82, what are the lengths of the three sides?

15. If a rectangle's length is 7 more than 4 times the width and the perimeter is 54 what are the dimensions of the rectangle?

16. If a cone has a volume of 338 cm^3 and a radius of 6 cm, how tall is it?

17. Find the missing variable for a parallelogram:

$A = 64\ in^2$
$h =$
$b = 12.6$ in

Solve.

4.3

Example:

$x + 4 - 5x = 7x + 1$	Combine like terms
$-4x + 4 = 7x + 1$ $+4x \qquad +4x$	Get all x's together by adding 4x to both sides
$4 = 11x + 1$ $-1 \qquad -1$	Subtract 1 from both sides
$3 = 11x$	
$\frac{3}{11} = x$	Divide both sides by 11

18. $5p + 12 = 33 - p$

19. $7n + 18 = 5(n - 2)$

20. $5x - 10 = 5x + 7$

21. $x - 7 = 15x$

22. $2x - 4(x-3) = -2x + 12$

23. $.07x = 13 - .12x$

24. $.7(3x - 2) = 3.5x + 1$

25. $.3x - 9 + 2x = 4x - 3$

26. $.4y = 78 + .4y$

27. $7(x - 5) - 3x = 4x - 35$

28. $9x - 4(x - 3) = 15x$

29. $2x - 3x + 7x = 9x + 8x$

Solve for the specified variable.

30. $p = fx + bn$ for f

31. $F = \frac{xf - xz}{2}$ for f

32. $M = 5t - 3p$ for t

33. $LSA = \pi r l$ for r

34. $y = mx + b$ for x

35. $\frac{3s - 4g}{7} = c$ for g

Preparation.

36. Find the final price of an object that is $200 but has 15% off.

37. Find the final amount of a savings account that has $170 and then has 15% interest added to it.

38. After reading some of 4.4, try to find out what the original price of an object was if the final price after 15% off was $85.

Answers:

1. x = 17

2. x = -6

3. m = $\frac{1}{7}$

4. t = -28

5. x = -$\frac{8}{3}$

6. x = 9

7. t = 0

8. x = -4

9. x = 13

10. x = 14

11. 65, 86

12. 151, 285

13. 129.9 cm^2

14. 21, 28, 33

15. w = 4, *l* = 23

16. h = 8.97 cm

17. *h* = 5.08 in

18. p = $\frac{7}{2}$ or 3.5

19. n = -14

20. no solution

21. x = -$\frac{1}{2}$

22. All real numbers

23. 68.42

24. x = -$\frac{12}{7}$ or -1.71

25. x = -3.53

26. No solution

27. All numbers

28. x = $\frac{6}{5}$ or 1.2

29. x = 0

30. $f = \frac{p-bn}{x}$

31. $f = \frac{2F+xz}{x}$

32. $t = \frac{M+3p}{5}$

33. $r = \frac{LSA}{\pi d}$

34. $x = \frac{y-b}{m}$

35. $g = \frac{7c-3s}{-4}$ or $\frac{3c-7s}{4}$

36. In class

37. In class

38. In class

Section 4.4

Applications:
Percents

Percents

If you scored 18 out of 25 points on a test, how well did you do. Simple division tells us that you got 72%. As a review, 18/25 = .72

If we break up the word "percent" we get "per" which means divide and "cent" which means 100. Notice that .72 is really the fraction $\frac{72}{100}$. We see that when we write is as a percent instead of its numerical value, we move the decimal 2 places. Here are some more examples to make sure that we get percents:

.73 = 73%
.2 = 20%
.05 = 5 %
1 = 100%
2.3 = 230%

The next reminder, before we start doing problems is that the word "of" often means "times". It will be especially true as we do examples like:

What is 52% of 1358?

All we need to do is multiply (.52)(1358)

which is 706.16

Sometimes however, it isn't quite that easy to see what needs to be done. Here are three examples that look similar but are done very differently. Remember "what" means "x", "is" means "=" and "of" means times.

What is 15% of 243?

x = .15(243)
x = 36.45

15 is what percent of 243?

15 = x (243)
.062 = x
6.2% = x

15 is 243% of what?

15 = 2.43x
6.17 = x

Once we have that down, we have the ability to solve tons of problems involving sales tax, mark-ups, and discounts. Here are **two examples**:

An item sells for $85.59 but is on sale at 20% off. What is the final price?

.2(85.59) = 17.12 amount of discount

85.59 – 17.12 subtract discount

$68.47 = final price

An item sold at $530 has already been marked up 20%. What was the price before the mark-up?

x + .2x = 530

original + 20% of original = final price

1.2x = 530
x = 441.67

Section 4.4 Exercises

4.2

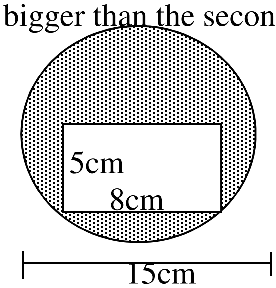
1. 45 is 12 more than 3 times a number. What is the number?

2. 25 less than 7 times a number is 108. What is the number?

3. Two numbers add to 251 and the second is 41 bigger than the first. What are the two numbers?

4. Two numbers add to 336 and the first is 124 bigger than the second. What are the two numbers?

5. Find the area of the shaded region:



6. I have created a triangular garden such that the largest side is 8m less than twice the smallest and the medium side is 12m larger than the smallest side. If the total perimeter of the garden is 104m, what are the lengths of the three sides?

7. If a rectangle's length is 5 more than 3 times the width and the perimeter is 58 mm what are the dimensions of the rectangle?

8. A total of $4.45 is made up of quarters, nickels, and dimes. The number of dimes is 2 more than three times the number of quarters. The number of nickels is 6 times the number of quarters. How many of each coin are there?

9. If a parallelogram has an area of 258.9 cm^2 and a base of 23.2 cm, how tall is it?

10. Find the missing variable for a trapezoid:

$A = 68$ ft^2
$b =$
$h = 4$ft
$B = 21$ft

Solve.

4.3

11. $7p + 13 = 33 - 4p$

12. $5n + 48 = 7n - 2(n - 2)$

13. $5x - 10 = 7(x - 2)$

14. $3x - 7 = 12x$

15. $5x - 7(x+3) = -2x - 21$

16. $.06x = 15 - .18x$

17. $.8(7m - 2) = 9.5m + 1$

18. $.2q - 7 + 2q = 3q - 5$

19. $12t = 45 + .4t$

20. $6(x - 5) - x = 5x - 20$

21. $9x - 2(x - 3) = 15x + 7$

22. $5x - 13x + x = 7x + 8x$

4.4

23. 18 is what percent of 58?

24. What is 87% of 54?

25. 34 is 56% of what?

26. What is 13% of 79?

27. 119 is 8% of what?

28. 23 is what percent of 74?

29. Original Price:$92.56
Tax: 7.3%
Final Price:

30. Original Price:
Discount: 40%
Final Price: $43.90

31. Original Price:
Tax: 5%
Final Price: $237.50

32. Original Price: $58.50
Discount: 30%
Final Price:

33. If the population of a town grew 21% up to 15,049. What was the population last year?

34. If the price of an object dropped 25% down to $101.25, what was the original price?

Preparation.

35. After reading some from Section 4.5, try to solve this equation.

$$\frac{x}{7} + \frac{13}{7} = \frac{15}{7} - \frac{2x}{7}$$

36. Solve.

$$\frac{x}{3} + \frac{13}{3} = \frac{15}{3} - \frac{2x}{3}$$

Answers:

1. x = 11
2. x = 19
3. 105, 146
4. 106, 230
5. 136.71 cm^2
6. 25m, 37m, 42m
7. w = 6mm, *l* = 23mm
8. 5 quarters, 17 dimes, 30 nickels
9. 11.16 cm
10. 13ft = b
11. p = $\frac{20}{11}$
12. No solution
13. x = 2
14. x = $-\frac{7}{9}$
15. All numbers
16. x = 62.5
17. m = $-\frac{2}{3}$
18. q = -2.5
19. t = 3.879
20. No solution
21. x = $-\frac{1}{8}$
22. x = 0
23. 31%
24. 46.98
25. 60.7
26. 10.27
27. 1487.5
28. 31%
29. \$99.32
30. \$73.17
31. \$226.19
32. \$40.95
33. 12,437
34. \$135
35. In class
36. In class

Assignment 4.4

Section 4.5

Linear Equations w/ fractions

The one other thing that might throw you off is when you see a bunch of ***fractions*** in the problem. Not to worry, remember that you have power to do anything you want to the equation.

For example:

$\frac{3}{8}x-\frac{5}{8}=\frac{7x}{8}$ might be easier to look at if there weren't so many fractions in the way. Well, get rid of them. Multiply by 8 on both sides.

$^{(8)}\frac{3}{8}x-^{(8)}\frac{5}{8}=^{(8)}\frac{7x}{8}$ which makes it become:

$3x - 5 = 7x$ (not bad at all)
$-5 = 4x$
$-\frac{5}{4} = x$ Ta Da.

Worse example:

$\frac{2}{7}-\frac{x-3}{4}=5$ looks scary.

You have the ability to wipe out all of the fractions. Fractions are simply statements of division. The opposite of division is multiplication – and you have the power to multiply both sides of the equation by anything you want to. The question is, what will undo a division by 7 and by 4; the answer is multiplication by 28. Here is what it looks like:

1. Simplify

$$\frac{2}{7}-\frac{x-3}{4}=5$$

$$(28)\frac{2}{7}-(28)\frac{x-3}{4}=5(28)$$ (multiplying everything by 28)

$(4)2 - (7)(x - 3) = 140$ (28/7 = 4 and 28/4 = 7)

$8 - 7x + 21 = 140$ (Distribute the -7)

$-7x + 29 = 140$ (Combine numbers)

2. Subtract → $-7x = 111$ (Subtract 29 from both sides)

3. Divide → $x = -\frac{111}{7}$ (Not a nice looking answer, but it is right!)

Every problem can be boiled down to three steps:

Linear Equations

1. Simplify
    1. Parentheses
    2. Fractions
    3. Combine like terms
2. Add/Subtract
3. Multiply/Divide

Section 4.5 Exercises

4.2

1. 35 less than 7 times a number is 98. What is the number?

2. Two numbers add to 351 and the second is 71 bigger than the first. What are the two numbers?

Solve.

4.3

3. $7p + 12 = 33 - 4p$

4. $3n + 48 = 7 - 2(n - 2)$

5. $5x - 10 = 5(x - 2)$

6. $3x - 7 = 15x$

7. $5x - 7(x+3) = -2x + 12$

8. $.09x = 13 - .18x$

9. $.8(3x - 2) = 9.5x + 1$

10. $.2x - 7 + 2x = 3x - 5$

11. $12m = 70 + .4m$

12. $5(x - 5) - x = 4x - 20$

13. $9x - 4(x - 3) = 15x + 7$

14. $8x - 12x + x = 9x + 8x$

4.4

15. 85 is what percent of 39?

16. 85 is 54% of what?

17. What is 19% of 2,340?

18. What is 23% of 79?

19. 119 is 18% of what?

20. 43 is what percent of 174?

21. Original Price:$80.00
Tax rate:
Final Price: $85.60

22. Original Population: 8,824
Growth rate:
Final Population: 11,212

23. Original Price:$72.56
Tax: 7.3%
Final Price:

24. Original Price:
Discount: 30%
Final Price: $49.70

25. Original Price:
Tax: 5%
Final Price: $339.50

26. Original Price: $55.50
Discount: 40%
Final Price:

27. If the population of a town grew 31% up to 17,049. What was the population last year?

Assignment 4.5

28. If the grading for a course is weighted with 50% on tests, 30% on homework, 10% tutoring, and 10% attendance, what is a student's grade if he got 85% on his tests, 70% on homework, 5% in tutoring and 90% in attendance?

Solve.

4.5

Example:

$\frac{2}{3}(x+4) - 5 = \frac{1}{2}x + \frac{5}{3}$	
$(12)\frac{1}{3}(x+4) - 5^{(12)} = (12)\frac{1}{4}x + {}^{(12)}\frac{5}{6}$	Clear fractions by multiplying by 12
$4(x+4) - 30 = 3x + 10$	
$4x + 16 - 30 = 3x + 10$	Distribute through parentheses
$x - 14 = 10$	Combine, getting x to one side
$x = 24$	Add 14 to both sides

29. $\frac{7}{3}t - 5 = 19$

30. $-\frac{3}{8}(x - 7) = 5 + 3x$

31. $\frac{2}{3}x - 6 = 3 + \frac{1}{2}x$

32. $\frac{4}{5}x = 2x - \frac{5}{3}$

33. $\frac{3}{5}x - \frac{2}{5}(x-3) = \frac{1}{5}x + 3$

34. $\frac{3x+2}{7} = \frac{4x-1}{5}$

35 $.9(-4x - 5) = 2.5x + 6$

36. $.0005x + .0045 = .004x$

37. $\frac{x+7}{4} = 8 - \frac{5}{6}x$

Preparation.

38. Describe the best way to get rid of fractions in an equation.

Answers:

1. 19

2. 140, 211

3. $p = \frac{21}{11}$

4. $n = -\frac{37}{5}$ or -7.4

5. All numbers

6. $x = -\frac{7}{12}$

7. no solution

8. $x = 48.15$

9. $x = -.366$

10. $x = -2.5$

11. $m = 6.03$

12. no solution

13. $x = \frac{1}{2}$

14. $x = 0$

15. 218%

16. 157.4

17. 444.6

18. 18.17

19. 661.1

20. 24.7%

21. 7%

22. 27.1%

23. $77.86

24. $71.00

25. $323.33

26. $33.30

27. 13,015

28. 73% “C”

29. $t = \frac{72}{7}$

30. $x = -\frac{19}{27}$

31. $x = 54$

32. $x = \frac{25}{18}$

33. no solution

34. $x = \frac{17}{13}$

35. $x = -\frac{105}{61}$

36. $x = \frac{9}{7}$

37. $x = \frac{75}{13}$

38. In class

Summary of Linear Equations

Linear Equations

1 – Simplify
  - Parentheses
  - Fractions
  - Combine like terms

2 – Add/Subtract
3 – Multiply/Divide

Word Problems

D,V,P,E

Absolute Value equations

1 – Get absolute value alone
2 – Split with negative.
3 – Solve like normal.

Inequalities

1 – Act like equals.
2 – Switch direction when mult/div by a negative
3 – Graph correct side.

Absolute value inequalities

1 – Split w/neg
2 – Solve as normal inequalities
3- OR is either one or the other, AND means both at the same time

Special Cases

No solution – an impossible equation like $0=1$, $5>7$ or $|x|<-4$
All numbers – always true equation like $0=0$, $5<7$ or $|x|>-4$

Chapter 4 Review Exercises (1)

Solve.

4.1

1. $5\left(\frac{3x-1}{5}-2\right)=70$

2. $3\left(\frac{-6x+4}{2}+3\right)-5=19$

3. $-4 - 9m = -22$

4. $\frac{6}{7}t = -24$

5. $19 = 3x - 7$

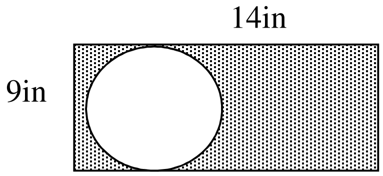
6. $\frac{5x-7}{3}=-9$

4.2

7. 84 is 6 more than 3 times a number. What is the number?

8. Two numbers add to 438 and the first is 74 bigger than the second. What are the two numbers?

9. Find the area of the shaded region:



10. If a rectangle's length is 7 more than 4 times the width and the perimeter is 194 mm, what are the dimensions of the rectangle?

11. Find the missing variable for a rectangle:

P = 48.3 ft
w = 7.2 ft
l =

12. Find the missing variable for a sphere:

SA = 800 in^2
r =

Solve.

4.3

13. $7p + 12 = 13 - 7p$

14. $4n + 68 = 7 - 2(n - 2)$

15. $7x - 10 = 5(x - 2)$

16. $9x - 4 = 15x$

17. $8x - 7(x+3) = x - 21$

18. $.18x = 13 - .20x$

Solve for the specified variable.

19. $\frac{2s-at^2}{2t}=V$ for s

20. $r = \frac{I}{pt}$ for p

21. $d=\frac{LR_2}{R_2+R_1}$ for R_1

22. $\frac{9s-5g}{11}=c$ for *s*

4.4

23. 14 is what percent of 68?

24. What is 37% of 754?

25. 119 is 18% of what?

26. 27 is what percent of 74?

27. Original Price:$192.56
Tax: 7.3%
Final Price:

28. Original Price:
Discount: 35%
Final Price: $43.90

29. If the price of a meal after a 20% tip was $28.80? What was the price of the meal before the tip was added?

30. If the price of an object dropped 15% down to $59.50, what was the original price?

Solve.

4.5

31. $\frac{7}{3}t - 2 = 19 + 5t$

32. $-\frac{3}{4}(x - 4) = 5 + 2x$

33. $\frac{1}{6}x - 4 = 3 + \frac{3}{10}x$

34. $\frac{5}{2}(-4x - 2) = \frac{3}{4}x + 6$

35. $\frac{x-5}{3} = \frac{5x+8}{6}$

36. $\frac{x+7}{14} = 6 - \frac{3}{7}x$

Answers:

1. $x = 27$

2. $x = -1$

3. $m = 2$

4. $t = -28$

5. $x = \frac{26}{3}$

6. $x = -4$

7. 26

8. 182, 256

9. 62.38 in^2

10. 18mm X 79mm

11. $l = 16.95$ ft

12. 7.98in

13. $p = \frac{1}{14}$

14. $n = -9.5$

15. $x = 0$

16. $x = -\frac{2}{3}$

17. All numbers

18. $x = 34.21$

19. $s = \dfrac{2Vt + at^2}{2}$

20. $p = \dfrac{I}{rt}$

21. $R_1 = \dfrac{LR_2 - dR_2}{d}$

22. $s = \dfrac{11c + 5g}{9}$

23. 20.6%

24. 278.98

25. 661.1

26. 36.5%

27. \$206.62

28. \$67.54

29. \$24

30. \$70

31. $t = -\frac{63}{8}$

32. $x = -\frac{8}{11}$

33. $x = -52.5$

34. $x = -\frac{44}{43}$

35. $x = -6$

36. $x = 11$

Chapter 4 Review Exercises (2)

1. Create a visual chart of all of the methods, formulas, and examples from studying how to solve these linear equations.

Solve.

4.1

2. $-\frac{2}{9}m = 24$

3. $9\left(\frac{8-6x}{4}+6\right)+5=-31$

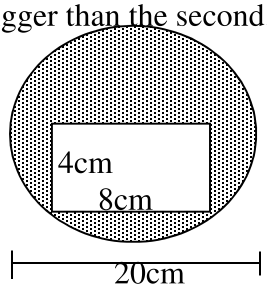
4. $\frac{8x-5}{3}=33$

4.2

5. 13.2 less than 7 times a number is 18.8. What is the number?

6. Two numbers add to 336 and the first is 24 bigger than the second. What are the two numbers?

7. Find the area of the shaded region:



8. I have created a triangular garden such that the largest side is 6m less than twice the smallest and the medium side is 15m larger than the smallest side. If the total perimeter of the garden is 105m, what are the lengths of the three sides?

9. If a parallelogram has an area of 158.9 cm^2 and a base of 23.2 cm, how tall is it?

10. Find the missing variable for a trapezoid:

$A = 76\ ft^2$
$b =$
$h = 4ft$
$B = 23ft$

Solve.

4.3

11. $7p + 12 = 12 + 7p$

12. $9n + 48 = 7n - 2(n - 2)$

13. $7x + 18 = 9(x - 3)$

Solve for the specified variable.

14. $d = \frac{pM(f-t)}{R}$ for p

15. $d = \frac{pM(f-t)}{R}$ for R

4.4

16. 45 is what percent of 39?

17. 25 is 44% of what?

18. What is 59% of 2,340?

19. What is 83% of 79?

20. Original Price:
Tax: 5%
Final Price: $359.50

21. Original Price: $55.50
Discount: 20%
Final Price:

22. If the population of a town grew 11% up to 17,046. What was the population last year?

23. If the price of an object dropped 15% down to $62.90, what was the original price?

Solve.

4.5

24. $\frac{7}{3}t - 8 = 4 + 7t$

25. $-\frac{3}{7}(m - 12) = 3m + 6$

26. $\frac{5}{6}x - 8 = 7 + \frac{7}{8}x$

27. $.13(-2x + 2) = .05x + 7$

28. $\frac{x-7}{4} = \frac{5x+3}{10}$

29. $\frac{x+7}{14} = 1 - \frac{4}{7}x$

Answers:

1. It better be good.

2. m = -108

3. x = 8

4. x = 13

5. $p = \frac{dR}{M(f-t)}$

6. $R = \frac{pM(f-t)}{d}$

7. $\frac{32}{7}$

8. 156, 180

9. 282.16cm^2

10. 24m, 39m, 42m

11. 6.85cm

12. b = 15

13. All numbers

14. n = -11

15. x = $\frac{45}{2}$ or 22.5

16. 115.4%

17. 56.82

18. 1380.6

19. 65.57

20. $342.38

21. $44.40

22. 15,357

23. $74

24. t = -$\frac{18}{7}$

25. m = -$\frac{1}{4}$

26. x = -360

27. x = -21.74

28. x = -$\frac{41}{5}$ or -8.2

29. x = $\frac{7}{9}$